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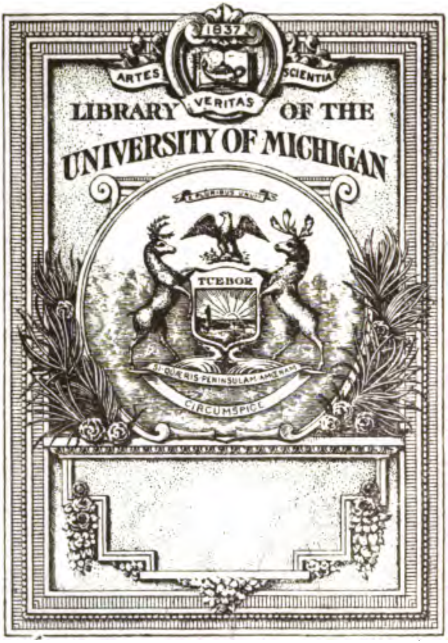
# ELECTRICAL MEASUREMENTS AND METER TESTING

MORETON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# Electrical Measurements and Meter Testing

A Book in Plain English for the Student and Practical Man—Fundamental Theory, Practical Applications and Examples

By
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# PREFACE

The text matter contained in this book has been prepared and arranged by the author with the requirements in mind of the practical men who are desirous of obtaining a working knowledge of the subject of Electricity as applied to Electrical Measurements and Meter Testing, but who are unable to take a complete course in Electrical Engineering.

It is the opinion of the author that such individuals should have a clear and reasonably thorough understanding of the fundamental principles of Electricity in order to appreciate the application of these principles in practice; therefore the first five chapters are devoted to a treatment of these principles.

Chapters six to thirteen, inclusive, treat of electrical measurements, and the construction, operation, and calibration of the instruments used in making such measurements.

Examples and their solutions are given throughout the book. These serve to illustrate the application of certain relations and principles, and thus give one an opportunity to get a clearer understanding of their true significance and value in practice.

The author expresses his thanks to the various manufacturing companies who have been so liberal in giving information and supplying cuts.

THE AUTHOR.

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# ELECTRICAL MEASUREMENTS AND METER TESTING

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## CHAPTER I

### DIRECT-CURRENT ELECTRIC CIRCUIT

*Electricity.*—The exact nature of electricity is not known, yet the laws governing its action, under certain definite conditions, are well understood, just as the laws of gravitation are known, although we cannot define the constitution of gravity. Electricity is neither a gas nor a liquid; its behavior sometimes is similar to that of a fluid, so that it is said to flow through a circuit. It must be understood that this expression of flowing does not mean there is an actual movement of something in the wire, similar to the flow of water in a pipe, when it possesses electrical properties, but is simply a convenient expression for the phenomena involved. Many an electrical problem can be easily understood by a comparison with a similar hydraulic problem, where the relation of the various qualities and the results are apparent under the existing conditions; and, on account of the similarity of the two, the hydraulic analogy will often be used to illustrate what is supposed to take place in the electrical circuit.

*Electrical Circuit.*—The electrical circuit is the path in which the electricity moves. A study of the electrical circuit forms a very appropriate beginning for the subject of electrical measurements, because a thorough knowledge of the properties of electrical circuits and the quantities associated with them is essential in order to make such measurements.

Electrical circuits are of various forms and are made for many different purposes, but all possess to a greater or less extent the same properties, and involve the same electrical quantities. Suppose, for example, an electrical heater which is operated by current from a small generator, as shown in Figure 1. This simple combination forms an electrical circuit which is typical of all electrical circuits. It contains a source of electrical energy—the generator; an energy transforming device—the electric heater; and the necessary connecting material—wires and switch. All electrical circuits are closed on themselves and, like the circumference of a circle, have neither beginning nor end.

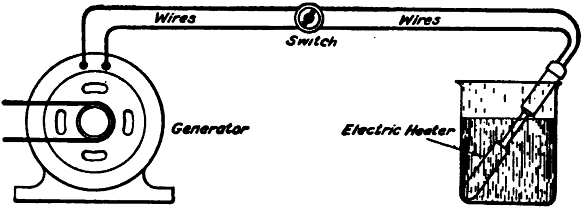


Figure 1.—Typical Electrical Circuit.

*Hydraulic Analogy of the Electrical Circuit.*—A brief consideration of a simple hydraulic circuit and the relation of the quantities involved may be an aid to a clearer understanding of the electrical circuit and the quantities associated with it. It must be understood that there is no similarity between the water and the electricity only in regard to action. An hydraulic circuit, shown in Figure 2, consists of a pump *P*, which is connected by pipes of different sizes and lengths to a water motor *M*. A valve *V* is placed in one of the pipes so as to control the flow of water; and pressure gauges $G_1$, $G_2$, etc., are connected at various points along the circuit, as shown in the figure. When the valve is closed, no water will flow in the pipes, and all the gauges will indicate the same pressure. If the valve is opened, the pressure produced by the pump will cause a flow of water through the pipes and water motor, and the indications of the gauges will all be different, their indications decreasing as you pass along the circuit, starting from the outlet of the pump, which is marked *O* in the figure. The difference in the pressure indicated by any two gauges is a measure of the pressure required to cause the water to flow between the two points where the gauges are connected to the circuit. If the pipes offered no opposition to the flow of the water through them, then the gauges connected to the ends of the different sections would all indicate the same pressure, regardless of whether there be a flow of water or not. Anything which will increase the opposition offered by a given section of the circuit, such as increasing the length, decreasing the area of the opening in the pipe, or attaching obstructions to the inner surface of the wall of the pipe, will cause an increase in the difference between the readings of the two gauges connected

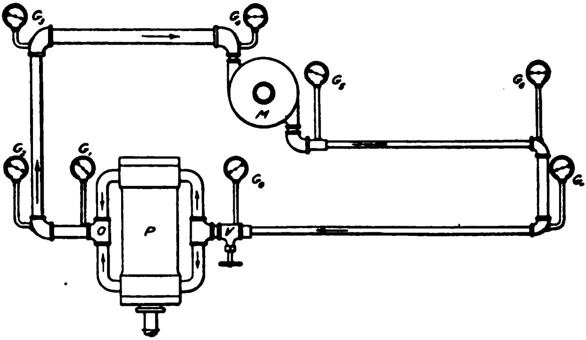


Figure 2.—Hydraulic Circuit.

to opposite ends of the section, the quantity of water passing through the pipe in a given time being maintained constant by adjusting the pressure produced by the pump. If the difference between the readings of two gauges connected to the ends of a section of pipe be maintained constant by adjusting the pump pressure and the opposition offered by the section of pipe be increased the quantity of water passing through the section of the pipe in a given time will be decreased; similarly the quantity of water passing through the section of the pipe will be increased if the opposition offered by the pipe is decreased.

The difference between the readings of the two gauges connected before and after the water motor represents the pressure required to overcome the opposition offered by the circuit through the motor and the pressure required to cause the revolving portion to turn. If there is no load on the motor, very little pressure will be required to operate it; but with an increase in load, there must be an increase in pressure acting on the motor if the speed is to be maintained constant. The action of the water motor in this case corresponds to the constant-current series motor in the electrical circuit.

In such a circuit as the one shown in Figure 2, the total pressure supplied by the pump will be distributed over the various sections in proportion to their opposition. Any change in the opposition of a section will result in a change in the difference between the readings of the two gauges connected to its opposite ends. If the total pressure acting on the circuit remains constant, an increase or decrease in pressure on one section due to a change in its opposition will result in a decrease or increase in pressure over the remaining sections. If the quantity of water flowing through the circuit is maintained constant and the opposition of a given section is increased, the total pressure must be increased; or if the resistance of a given section is decreased, the total pressure must be decreased, otherwise the flow of water would be changed.

The opposition offered by the pipe to the free flow of water through it is called its *resistance,* and it will depend upon the size of the opening in the pipe, length of the pipe, nature of the inner surface, any obstruction that might be in the pipe, such as sieves, sand, and gravel, etc.

The quantity of water passing a given cross-section in the pipe in a unit of time is called the *current.* For example, if there was a uniform flow in the pipe of 300 gallons in one minute, the current would be 300 gallons per minute, or 5 gallons per second.

The three quantities—pressure, current, and resistance—are related in a very simple way, as seen from the above example, and this relation expressed in the form of an equation is:

$$\text{Current} = \frac{\text{pressure}}{\text{resistance}}$$

It must be remembered that this equation does not hold in its strictest sense for the hydraulic problem, but will serve to illustrate the relation between the quantities associated with the electrical circuit which are discussed in the following section.

*Electrical and Hydraulic Circuits Compared.*—The electrical circuit shown in Figure 3 is very similar to the hydraulic circuit shown in Figure 2. In the electrical circuit there is a source of electrical pressure—the generator, an electrical transforming device—the lamps, which corresponds to the water motor; the wires, which serve to conduct the electricity as the pipes conduct the water; and a switch, which controls the circuit in a manner similar to the control of the hydraulic circuit by means of the valve. The difference in electrical pressure between any two points *A* and *B* on the electrical circuit is a measure of the pressure required to cause a given quantity of electricity to pass from one of the points to the other in a given time. This difference in pressure will depend upon the opposition offered by the circuit between the two points and the quantity of electricity passing between them in a given time. With an increase in the opposition offered by the electrical circuit there must be an increase in pressure in order to maintain constant the quantity flowing per unit of time. If, on the other hand, the pressure remains constant and the opposition to the flow increases or decreases, there will be a decrease or increase in the quantity of electricity passing a given cross-section of the circuit in a given time.

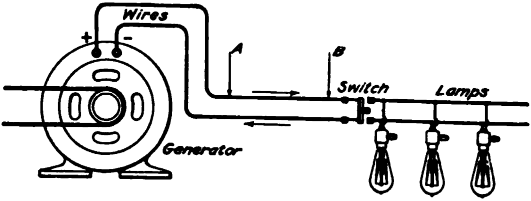


Figure 3.—Electrical Circuit.

The opposition offered by the material composing the electrical circuit to the flow of electricity through it is called the *resistance of the circuit,* and it is measured in a unit called the *ohm.* (See Chapter VI for a complete discussion of resistance.)

The rate at which the electricity moves in the electrical circuit is called the *current.* The unit quantity of electricity is the *coulomb,* and if the rate of flow is such that one coulomb passes a given cross-section in one second, there is said to be a current of one ampere. (See Chapter VII for a complete discussion of current.)

Since every part of the electrical circuit offers resistance to the flow of electricity, there must be some force, or pressure, to overcome this resistance and maintain the current. This force is the electromotive force, or electricity moving force, and it is measured in a unit called *volt*. (See Chapter VII for a complete discussion of electrical pressure.)

There are a number of ways of generating an electromotive force, e.m.f. (abbreviated), the two most common being, perhaps, the chemical generation in the primary cell and the mechanical generation, by the process of electromagnetic induction, in the generator. It must be clearly understood that electromotive force does not create electricity but simply imparts energy to it, as a mechanical force may impart energy to a body by producing motion. Electromotive force can exist without producing a current, just as a mechanical force can exist without producing motion.

*Conductors and Insulators.*—All materials will conduct electricity if sufficient electrical pressure be applied, but the degree to which some materials conduct, as compared to others, is quite different. For example, the opposition offered by a piece of copper to the flow of electricity through it is much less than a piece of glass of equal dimensions. As a result of this difference in conducting properties, materials are grouped in two main classes: conductors and insulators. An insulator then is a material which offers a high opposition to the flow of electricity through it, as compared to another material called a conductor. The two terms are only relative and there is no marked division between conductors and insulators.

*Electromotive Force and Potential Difference.*—The electromotive force in any electrical circuit is the total generated electrical pressure acting in the circuit, while the potential difference is the difference in electrical pressure between any two points in the circuit. The electricity as it passes along a conductor loses its energy; hence, between any two points in the circuit there will be a difference in energy or potential possessed by the electricity. For example, in overcoming the resistance of the circuit between the points *A* and *B*, Figure 3, the electricity will lose some of its energy and will, therefore, have less potential at *B* than at A, or, in other words, there is a difference in potential between the points *A* and *B*. This difference in pressure in the electrical circuit is analogous to the difference in pressure between two points along the hydraulic circuit. This difference in potential, p. d. (abbreviated), is due to the current through the resistance for, when the current stops, the difference in potential will no longer exist. The potential difference between any two points is measured in the same unit as electromotive force, namely, the volt.

The difference in electrical pressure between two points on an electrical circuit is often called the *voltage,* or *copper drop,* between the points.

*Ohm's Law.*—Current, electrical pressure, and resistance are always present in every active circuit and there is a simple, but very important, relation connecting them. This relation was discovered by Dr. G. S. Ohm about 1827 and has, as a result, been called *Ohm's Law.* Dr. Ohm discovered by experiment that the difference in electrical pressure between any two points on an electrical circuit is strictly proportional to the current, all other conditions remaining constant. This discovery expressed in the form of an equation would read as follows:

$$\text{electrical pressure} = \text{constant} \times \text{electrical current}$$

The constant in the above equation was found to change in value directly as the resistance or opposition of the circuit, so, by a proper selection of the value of the units in which to measure the electrical pressure, electrical current, and electrical resistance, the above equation may be rewritten as follows:

$$\text{electrical pressure} = \text{electrical resistance} \times \text{electrical current}$$

Substituting the units in which the above quantities are measured, the equation may be written as follows:

$$\text{volts} = \text{ohms} \times \text{amperes}$$

The symbols usually employed in representing volts, ohms, and amperes are $E$, $R$, and $I$, respectively. Substituting these symbols for the quantities in the above equation gives

$$E = R \times I \qquad \text{(a)}$$

Other forms in which the above equation may be written are as follows:

$$\text{amperes} = \frac{\text{volts}}{\text{ohms}}$$

$$= \frac{E}{R} \qquad \text{(b)}$$

or

$$\text{ohms} = \frac{\text{volts}}{\text{amperes}}$$

$$R = \frac{E}{I} \qquad \text{(c)}$$

All of the above relations hold true for all or any part of a circuit composed of metals and electrolytes, but it does not seem to be true for gases under certain conditions, nor for insulators.

*Examples.*—1. The resistance of a circuit is 50 ohms. What current will a pressure of 110 volts produce in the circuit?

*Solution.*—Substituting the values of the resistance and pressure in equation (b) as given above, we have

$$I = \frac{110}{50} = 2.2 \text{ amperes}$$

**2. What pressure is required to produce a current of 22 amperes in a circuit having a resistance of 6 ohms?**

***Solution.*—Substituting the values of current and resistance in equation (a) as given above, we have**

$$E = 6 \times 22 = 132 \text{ volts}$$

*Electrical Work and Energy.*—In general terms, a force is that which produces or tends to produce a change in motion; thus a force must always be applied to a body to cause it to move, and a force must be again applied to cause it to come to rest. A force does not always produce motion, but only tends to produce it, as when you push on a brick wall you apply muscular force, but there is no motion of the wall. There are a number of different kinds of force, the most common of which are as follows: gravitational force, as a result of which all bodies fall from a higher to a lower level; mechanical force, which may be produced by the expansion of steam in the engine cylinder; and electrical force, or that force which produces or tends to produce a movement of electricity. Electrical force is commonly produced in the primary battery or by an electrical generator.

When a force overcomes a certain resistance, work is done; or work is the result of the action of a force. It must be understood, however, that a force may exist without any work being done. Thus, a generator may be generating an electrical force, but if it is not sufficient to overcome the electrical resistance between the terminals of the machine, no current is produced and, therefore, the generator is not doing any work. If, however, the armature of the generator be made part of a closed electrical circuit, a current will be produced and, as a result, the generator will do work.

Energy is the ability to do work. Electrical energy, then, is the ability to do electrical work, but it is measured in the same units as work, it being numerically equal to the work done. For example, if in the hydraulic circuit, Figure 2, a pressure of 100 pounds is acting on the entire circuit and 25 pounds of water pass through the circuit, then the work done is equal to the product of the pressure and the quantity moved, or

$$100 \times 25 = 2500 \text{ foot-pounds}$$

The relation between quantity and pressure in the electrical circuit is identical to the above relation for the hydraulic circuit. For example, if there is an electrical pressure of 110 volts acting on an electrical circuit and 10 coulombs of electricity pass through the circuit, then the work done will be equal to the product of the pressure and the quantity moved.

The energy of the water in the hydraulic circuit is increasing if it is passing from a given pressure to a higher pressure and, likewise, it is decreasing if it is passing from a given pressure to a lower pressure. In the hydraulic circuit, Figure 2, the energy of the water is decreasing as it passes from the outlet *O* around the circuit and again returns to the pump, where energy is again imparted to it by the action of the pump or its pressure is raised. The loss in energy between any two points at various positions along the circuit will bear the same relation to each other as exists between the corresponding differences in pressure. Thus, if the difference in pressure as indicated by gauges $G_4$ and $G_5$ is 80 pounds and the total pressure produced by the pump is 100 pounds, then the work done by the pump will be divided between the water motor *M* and the remainder of the circuit in the ratio of 80 to 20 units. It is always desirable to have as large a part possible of the total pressure produced by the source act on the useful part of the circuit, for the remaining pressure does no useful work.

The energy possessed by a certain quantity of electricity is increasing when it is passing from a lower to a higher electrical level and decreasing when it is passing from a higher to a lower electrical level. The electrical level of a point is always measured with reference to some other point chosen arbitrarily as a reference point and it is expressed in the same unit as electrical pressure. A voltmeter, having one terminal connected to the reference point and the other connected to the point whose electrical level is to be determined, will give an indication which may be called the *electrical level of the point.* In electrical circuits it is not the electrical level of any one point that is of service, but it is the difference in electrical level between any two points, just as in the hydraulic circuit it is the difference in pressure between two points that is of greatest use rather than the pressure at any one point.

*Unit of Electrical Work or Energy.*—The joule, the unit of electrical work or energy, is the amount of work done in raising one coulomb of electricity through a difference in electrical level of one volt. Work is always independent of time, namely, it will require the same amount of work to raise a given quantity of matter a given height in feet in one hour as would be required to raise it the same height in two hours.

The name for the mechanical unit of work is usually a compound word in which one of the parts is a force unit and the other part a distance unit. The unit most commonly used is the *foot-pound*—the pound being the unit of force and the foot the unit of distance.

The quantity of electricity in coulombs flowing through an electrical circuit in a given time is equal to the product of the current in amperes and the time in seconds. Hence, in order to compute the work done in an electrical circuit, the current in amperes must first be multiplied by the time in seconds to obtain the quantity in coulombs, and this product in turn multiplied by the electrical pressure through which the quantity of electricity is raised. Work and energy are usually represented by the symbol $W$.

*Example.*—Determine the work done or the energy expended by a source of electrical pressure of 110 volts in producing a current of 10 amperes for 50 seconds.

*Solution.*—The quantity in coulombs of electricity moved is equal to the product of the current in amperes and the time in seconds, or

$$10 \times 50 = 500 \text{ coulombs}$$

The work done, or energy expended, in joules is equal to the product of the quantity in coulombs and the difference in electrical pressure in volts through which it is moved, or

$$500 \times 110 = 55{,}000 \text{ joules}$$

The above operations are usually combined in a single equation as follows:

$$\text{joules} = \text{volts} \times \text{amperes} \times \text{time (in seconds)}$$

*Electrical Power.*—Power is the rate of doing work, or the rate at which energy is expended. The unit of electrical power is a unit of electrical work performed in a unit of time, or a joule per second, and it is called the *watt*. Power is represented by the symbol $P$.

$$\text{electrical power} = \frac{\text{electrical work}}{\text{time}}$$

$$P = \frac{W}{t}$$

Substituting the value of the electrical work, as given in terms of current, pressure, and time in the previous section, in the above equation, we have

$$\text{electrical power} = \frac{\text{volts} \times \text{amperes} \times \text{time}}{\text{time}}$$

$$\text{watts} = \text{volts} \times \text{amperes}$$
$$P = EI$$

One watt, therefore, equals a current of one ampere under a pressure of one volt, or any combination of volts and amperes whose product is unity. The above equation for power may be rewritten so as to give the pressure in terms of power and current, and the current in terms of power and pressure, as follows:

$$\text{volts} = \frac{\text{watts}}{\text{amperes}}$$

$$E = \frac{P}{I}$$

and

$$\text{amperes} = \frac{\text{watts}}{\text{volts}}$$

$$I = \frac{P}{E}$$

*Examples.*—1. A direct-current generator supplies 25 amperes to a 110-volt motor. How many watts does the motor consume?

*Solution.*—Substituting directly in the above equation, which gives the value of the power in watts, in terms of current and pressure, we have

$$P = 110 \times 25 = 2{,}750 \text{ watts}$$

2. A certain group of incandescent lamps take 1,200 watts from a 115-volt direct-current circuit. What current do the lamps take?

*Solution.*—Substituting directly in the above equation, which gives the value of the current in amperes, in terms of the power and pressure, we have

$$I = \frac{1{,}200}{115} = 10.4 + \text{amperes}$$

3. What pressure is required to operate a 500-watt electrical heater if it takes a current of 2 amperes?

*Solution.*—Substituting directly in the above equation, which gives the pressure in volts, in terms of the power and current, we have

$$E = \frac{500}{2} = 250 \text{ volts}$$

*Mechanical and Electrical Horsepower.*—The horsepower any machine is developing is equal to the foot-pounds of work done per minute divided by 33,000, or the foot-pounds of work done per second, divided by 550.

One watt is equal to .7376 foot-pounds per second, or one foot-pound per second is equal to 1.356 watts. Since the mechanical horsepower is equal to 550 foot-pounds per second, an electrical equivalent rate of doing work would be

$$550 \div .7376 = 746 \text{ watts} = 1 \text{ electrical horsepower}$$

Hence, to change from mechanical horsepower to electrical units multiply by 746, or

$$\text{watts} = \text{horsepower} \times 746$$

To change power in watts to horsepower divide by 746, or

$$\text{horsepower} = \text{watts} \div 746$$

*Example.*—What is the horsepower output of a generator when it is delivering a current of 120 amperes at a pressure of 110 volts?

*Solution.*—The output in watts is equal to the product of the current and pressure, or

$$\text{output} = 120 \times 110 = 13{,}200 \text{ watts}$$

and the output, in watts, divided by 746 is the output in horsepower, or

$$\text{output} = 13{,}200 \div 746 = 17.6 + \text{horsepower}$$

*Larger Units of Work or Energy.*—The joule is a small unit of electrical work or energy and, as a result, larger units are usually used in practice, such as the watt-hour and kilowatt-hour. One watt expended for one hour is one watt-hour, and 1000 watts, or a kilowatt, expended for one hour is a kilowatt-hour. Work and energy may be measured in any unit which is a combination of a power and time unit. The watt-hour is equivalent to 3600 watt-seconds or 60 watt-minutes.

$$\text{Watt-hours} = \text{watts} \times \text{hours}$$

$$\text{Kilowatt-hours} = \text{kilowatts} \times \text{hours}$$

The dials of the watt-hour meters used by the central station companies usually record the energy supplied to the consumer for lighting or power purposes in watt-hours or kilowatt-hours.

*Example.*—A certain group of lamps takes a current of 30 amperes from a 110-volt circuit. What will it cost to operate these lamps for 6 hours if the central station company charges 10 cents per kilowatt-hour?

*Solution.*—The power in watts taken by the lamps is equal to the product of the pressure and current, or

$$\text{power} = 110 \times 30 = 3{,}300 \text{ watts}$$

The energy input to the lamps in watt-hours is equal to the product of the power in watts and the time in hours, or

$$\text{energy input} = 3{,}300 \times 6 = 19{,}800 \text{ watt-hours}$$

The input in kilowatt-hours is equal to the above divided by 1,000, or

$$19{,}800 \div 1{,}000 = 19.8 \text{ kilowatt-hours}$$

The cost of 19.8 kilowatt-hours at 10 cents per kilowatt-hour will be

$$19.8 \times 10 = 198 \text{ cents, or } 1.98 \text{ dollars}$$

*Connecting Electrical Pressure in Series and Parallel.*—If two pumps $P_1$ and $P_2$ be connected as shown in Figure 4, they are said to be in series. The total pressure produced by the two will be equal to the algebraic sum of the pressures produced by the respective pumps. If both pumps are producing a pressure in the same direction, the total pressure is equal to the sum of the individual pump pressures; while, if the pumps are producing pressure in the opposite directions, the total pressure, or effective pressure, will be equal to the difference between the individual pump pressures. In general, the resultant or effective pressure in such a circuit is equal to the difference between the sum of the pressures acting in one direction and the sum of the pressures acting in the opposite direction, and the direction of this effective pressure will correspond to that of the larger sum.

Likewise, if several electrical pressures be connected in series—as indicated in Figure 5, which represents several cells $B_1$, $B_2$, and $B_3$—the effective electrical pressure is equal to the sum of the individual electrical pressures when they all act in the same direction around the circuit. If, however, some of the pressures act in one direction and some in the other, the effective pressure is equal to the difference between the sum of the pressures acting in one direction and the sum of the pressures acting in the reverse direction, and the direction of this effective pressure will correspond to that of the larger sum.

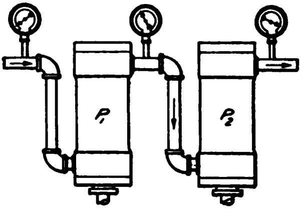


Figure 4.—Pump in Series.

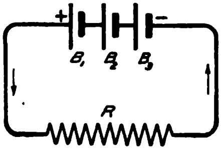


Figure 5.—Batteries in Series.

If two pumps $P_1$ and $P_2$ be connected as shown in Figure 6 and the water pumped by the two combines to form the current in the main part of the circuit, they are said to be connected in parallel or multiple. With such a connection the pressure over all the pumps is the same, but the water the combination is capable of supplying exceeds that of a single pump. The water supplied by each of the different pumps will be equal in volume if all the pumps are identical in construction and they are each producing the same pressure. A part of the pressure produced by each pump is used in causing the water to flow through the pump itself and, as a result, the pressure between the inlet and the outlet will be less than that actually produced by the pump when there is a current of water through the pump. When a number of pumps are connected in parallel, the total current will divide between the several pumps in such a manner that the pressure over all the pumps is the same. If one pump offers a greater opposition to the flow than another and their generated pressures are the same, it cannot supply as large a current as the other on account of its terminal pressure dropping faster per unit current.

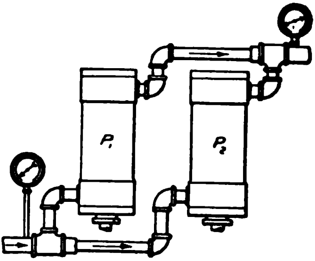


Figure 6.—Pumps in Parallel.

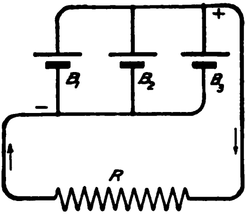


Figure 7.—Batteries in Parallel.

If several electrical pressures be connected as indicated in Figure 7, which represents several cells $B_1$, $B_2$, and $B_3$ in parallel, the pressure between the terminals of each branch of the circuit is the same for all branches, but the current the combination is capable of supplying is equal to the combined capacities of the several branches. If each branch is composed of the same number of cells and they all have the same characteristic—namely, the same electrical pressure and offer the same opposition to the current—the current in the different branches will be equal. When the cells have different characteristics, the division of current will be such that the difference in electrical pressure between the terminals of the several branches are equal.

The opposition offered by a cell to the flow of electricity through it is called its *internal resistance.* It is not constant in value, but depends upon the chemical condition of the cell, upon the current that the cell is supplying, and upon the duration of current supply, etc.

*Connecting Resistances in Series.*—When several resistances are so connected that there is only one path

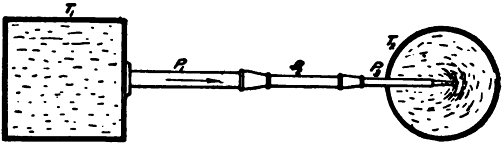


Figure 8.—Series Hydraulic Circuit.

for the current in passing through all of them, they are said to be connected *in series.* The three sections of pipe $P_1$, $P_2$, and $P_3$, Figure 8, are connected in series, and the total resistance of the combination is equal to the sum of the resistances offered by the several sections. Two resistances $R_1$ and $R_2$ are connected in series, Figure 9, and the combination is connected to the terminals of a battery composed of two dry cells. It is apparent the current has only one path from the positive to the negative terminal of the battery. These resistances may be of widely different values and composed of different materials, but the total resistance of the combination is equal·to the sum of the resistances of the various parts.

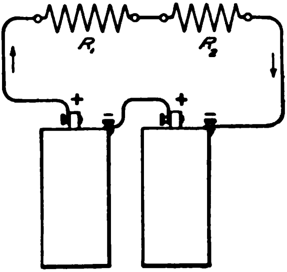


Figure 9.—Series Electrical Circuit.

There are four facts concerning every series circuit:

(1) The current is uniform throughout the series circuit.
(2) The difference in pressure over the different portions of a series circuit are proportional to resistances of these portions.
(3) The total resistance is the sum of the individual resistances.
(4) The effective electrical pressure is equal to the algebraic sum of all the pressures acting in the circuit.

*Connecting Resistances in Parallel or Multiple.*—When several resistances are so connected that there are as many paths for the current as there are connected resistances, the resistances are said to be connected *in parallel.* For example, in Figure 10 the three pipes $P_1$, $P_2$, and $P_3$ are each conveying water from the tank $T_1$ to the tank $T_2$—which is at a lower level—independent of each other and they are said to be connected in parallel. It is obvious that the current of water entering tank $T_2$ or leaving tank $T_1$ is equal to the sum of the currents in the three pipes. Now, since the same pressure is acting on each of the pipes and they are jointly carrying a greater current than either carries, their combined resistance must be less than that of any one of the pipes.

In computing the resistance of a number of different resistances connected in parallel, use is made of a property of the circuit called its *conductance*, which is numerically equal to one divided by the resistance.

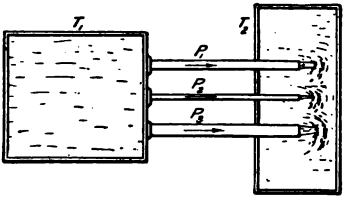


Figure 10.—Parallel Hydraulic Circuit.

In other words, the conductance of a circuit is its ability to conduct electricity and it is measured in a unit called the *mho*. The conductance of a number of resistances connected in parallel is equal to the sum of the conductances of the several resistances. If the combined resistance of a number of resistances in parallel be represented by $R$, and the individual resistances by $R_1$, $R_2$, $R_3$, etc., then we may write the following equation, which gives the resultant conductance:

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots\cdots$$

If the three resistances, Figure 11, have values of 4, 6, and 12 ohms, respectively, the conductance of the combination will be

$$\frac{1}{R}=\frac{1}{4}+\frac{1}{6}+\frac{1}{12}$$

$$\frac{1}{R}=\frac{6+4+2}{24}=\frac{12}{24}=\frac{1}{2}\text{ mho}$$

The resistance is equal to the reciprocal of the conductance, hence

$$R = 2 \text{ ohms}$$

Parallel or divided circuits are used quite frequently in connection with certain measuring instruments. For example, the moving system of ammeters oftentimes carries only a small part of the total current in the main circuit, as it is connected in parallel with a resistance, called a *shunt*, of much lower value. The values of the current in the various branches of a divided circuit have a definite relation to each other, which is the reverse of that existing between the resistances of the different branches—the lowest resistance branch carrying the larger current and the highest resistance branch carrying the smallest current.

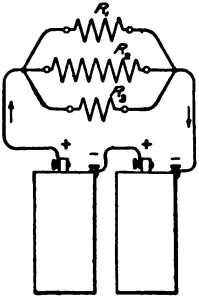


Figure 11.—Three Resistances in Parallel.

*Combination of Series and Parallel Connection.*—A number of resistances may be connected in such a way that some of them are in parallel with others, or some of them may be in series with others, or a combination of series and parallel connections may be formed. Six different resistances are shown connected together in Figure 12 and the combination in turn to the terminals of a two-cell battery. The resistances, for convenience, will be referred to as $R_1$, $R_2$, $R_3$, $R_4$, $R_5$, and $R_6$, respectively, and let us assume they have values of 5, 10, 6, 5, 7, and 6 ohms each. It will be seen, from an inspection of the scheme of connections, that the resistances $R_1$ and $R_2$ are in series and form one path between the points $A$ and $B$; that the resistances $R_4$ and $R_5$ are in series with each other and the combination is in parallel with the resistance $R_6$ and that the combination of $R_4$, $R_5$, and $R_6$ is in series with the resistance $R_3$ and forms another path between the points $A$ and $B$.

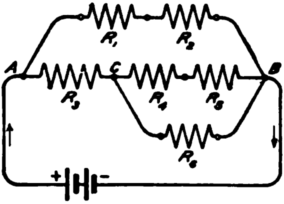


Figure 12.—Combination of Series and Parallel Circuits.

Let $B$ represent the resistance of the upper branch between $A$ and $B$; $B_2$ the resistance of the upper branch between $C$ and $B$; $B_3$ the resistance of the two branches between $C$ and $B$; $B_4$ the resistance of the lower branch between $A$ and $B$; $R$ the total resistance between $A$ and $B$; then

$$B_1 = R_1 + R_2 = 5 + 10 = 15 \text{ ohms}$$

$$B_2 = R_4 + R_5 = 5 + 7 = 12 \text{ ohms}$$

$$\frac{1}{B_3} = \frac{1}{B_2} + \frac{1}{R_6} = \frac{1}{12} + \frac{1}{6} = \frac{1}{4} \text{ mho}$$

$$B_3 = 4 \text{ ohms}$$

$$B_4 = R_3 + B_3 = 6 + 4 = 10 \text{ ohms}$$

$$\frac{1}{R} = \frac{1}{B_4} + \frac{1}{B_1} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6} \text{ mho}$$

$$R = 6 \text{ ohms}$$

# CHAPTER II

## MAGNETISM, ELECTROMAGNETISM, AND ELECTROMAGNETIC INDUCTION

### MAGNETISM

*Magnetism and the Magnet.*—Certain bodies have the property of attracting to themselves pieces of iron and steel, and this property is called *magnetism,* while the bodies themselves are called *magnets.* The ancients discovered that certain black stones, found at Magnesia in Asia Minor, possessed the property of attracting pieces of iron, and they called the bodies magnets. The magnet found by the ancients is an ore of iron, and its chemical composition is $Fe_3O_4$. It is found in quite large quantities in different parts of the world, but not always in the magnetic state.

*Natural and Artificial Magnets.*—A natural magnet is a body possessing magnetic properties as found in nature, and no special treatment is necessary in order to impart magnetism to it.

An artificial magnet is a body possessing magnetic properties due to a special treatment, such as being placed in contact with a natural magnet or placed under the influence of another magnet or an electric current. Artificial magnets, however, may be made very strong in comparison to the natural magnets and they are very extensively used in the construction of electrical instruments and equipment. A natural magnet and an artificial magnet are shown in Figures 13 and 14, respectively.

*Poles of a Magnet.*—If a magnet, such as the one shown in Figure 14, be freely suspended or supported so it may turn in a horizantal plane, it will be found that one end will point approximately north and the other approximately south. A magnet, when it is used in this way, is called a *compass needle,* but it usually has its two ends pointed. The end of the magnet pointing approximately north has been called the *north-seeking,* or *north pole,* and it is usually designated by the letter *N*. The other end is called the *south-seeking,* or *south pole,* and it is usually designated by the letter *S*.

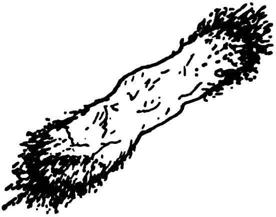


Figure 13.—Natural Magnet.

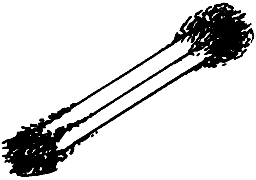


Figure 14.—Artificial Magnet.

If a magnet be dipped in a box of iron filings and then carefully removed, there will be a large mass of filings adhering to the ends or certain parts of the magnet, as shown in Figures 13 and 14. The magnetic force which holds the filings to the magnet is supposed to originate at what are called the poles of the magnet, which are, for simplicity in calculations, considered as being located at certain definite points in the magnets.

*Magnetic Substances.*—Any substance which is attracted by a magnet is called a *magnetic substance.* Iron and steel are the only two substances which are attracted to any great extent. Cobalt and nickel, however, are attracted by a strong magnet. Some substances, such as bismuth and antimony, are repelled instead of being attracted, but the action is very slight. It has been discovered in recent years that certain alloys can be made from non-magnetic substances, such as copper, aluminum, and magnesium, which are very decidedly magnetic. These combinations are known as *Heussler alloys*.

*Action Between Magnetic Poles.*—If the two poles of a magnet be presented in turn to the north pointing pole of a suspended magnet, it will be observed that there is an attraction between the poles with the one combination and a repulsion between them with the other combination. The polarity of the ends of each magnet should be determined before the experiment is performed and the poles properly marked. It will be observed, if the poles near each other are of the same polarity, that there will be a force of repulsion between them, as shown in Figure 15. If, on the other hand, the two poles near each other are not of the same polarity, there will be a force of attraction between them, as shown in Figure 16. The results of the above observations may be summarized

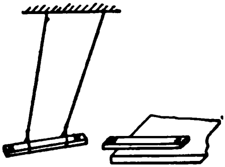


Figure 15.—Action of Like Poles on Each Other.

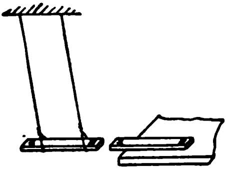


Figure 16.—Action of Unlike Poles on Each Other.

in a general law, as follows: *Like magnetic poles repel each other and unlike magnetic poles attract each other.*

*Magnetic Induction.*—Magnetism may be communicated to a magnetic substance by a magnet without their coming into actual contact. If a small rod of soft iron, which possesses no magnetism, be thrust into a pile of fine iron filings and withdrawn, no appreciable quantity of the filings will adhere to the rod. If, however, one end of the rod be thrust into the filings, and the pole of a magnet brought near the other end, and the rod then removed from the filings, keeping the pole of the magnet near its end but not touching, a large quantity of filings will cling to the end of the rod. When the pole of the magnet is removed, the filings will drop from the end of the rod, which indicates the magnetism possessed by the rod is due to the presence of the magnetic poles of the permanent magnet.

Several soft iron nails may be suspended from the end of a bar magnet, as shown in Figure 17, but if the magnet is carefully removed from the upper nail, all the others will fall away from each other, thus showing the nails are only magnets when they are under the influence of the bar magnet. Magnetism which is produced in bodies in the above ways by the presence of magnets either in contact or not, is called *induced magnetism*.

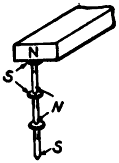


Figure 17.—Induced Magnetism.

If the polarity of the induced magnetism be tested by a magnet whose polarity is known, it will be found that the adjacent poles are unlike; that is, the end of the nail in contact with the bar magnet is of opposite polarity to the pole of the magnet. The end of the nail not in contact with the magnet is of the same sign as the pole of the magnet with which the nail is in contact. The process of producing magnetism in a body by placing it in contact with the magnet, or near it, is called *magnetic induction.*

The fact that a magnet attracts all magnetic materials is explained by means of magnetic induction, for the magnet first magnetizes the material by induction so that the near poles are unlike, and, as a result, there is a force of attraction. There is also a force of repulsion between the like poles, but since the unlike poles are closer together than the like poles, the force of attraction overbalances the force of repulsion and the material is drawn toward the magnet.

*Unit Magnet Pole.*—A unit magnetic pole is one of such a strength that when placed one centimeter from an equal and like magnetic pole there will be a force of repulsion of one dyne between them. The magnetic poles are considered as being concentrated at points and are called *point poles.*

The force in dynes acting between any two magnetic poles, of equal or unequal strengths $M_1$ and $M_2$ and of like or unlike polarity, when placed a distance of $d$ centimeters apart, is given by the equation

$$\text{force in dynes} = \frac{M_1 \times M_2}{d^2}$$

The force will be one of repulsion when the poles are of the same polarity, and one of attraction when they are of unlike polarity. In the above cases, the medium separating the two poles is assumed to be air, and the resultant action is different for mediums having a permeability other than unity.

*Lines of Force.*—If it were possible to completely separate a north magnetic pole from its corresponding south pole, and we were to place this north pole near the north pole of a bar magnet, it would move over to the south pole of the magnet along a curved path similar to the one shown in Figure 18. The path of the north pole is curved, due to the combined action of the north and the south poles of the bar magnet, the one repelling and the other attracting it; and the effect of each of these poles upon the independent pole is continuously changing, due to their distance from it changing.

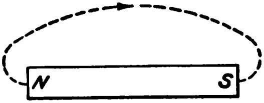


Figure 18.—Line of Force.

Any path which a north magnetic pole will take in going from a north pole to a south pole corresponds to a line of magnetic force.

*Magnetic Field.*—The entire region surrounding a magnet where a magnetic substance will be acted upon by a magnetic force, when placed in the region, is called a *magnetic field.* There are two properties possessed by every magnetic field which are of common interest, namely, the *direction* of the field and the *strength* of the field.

The direction of any magnetic field is determined by placing a unit north magnetic pole in the field and noting the direction of the magnetic force acting on the pole, which has been taken arbitrarily as the direction of the field. The direction of all magnetic fields produced by a magnet will always be from the north pole to the south pole.

The strength of a magnetic field at a given point is numerically equal to the force in dynes which would act on a unit north pole if placed at the point in question.

Magnetic fields are thought of as being composed of a large number of imaginary lines of force, and they are represented graphically by drawing lines about the magnet from the north to the south pole in such a manner that the direction of the lines at each point corresponds to the direction of the field at the point, and the number of lines per unit area perpendicular to their direction corresponds to the strength of the field at that particular point. The magnetic field surrounding a bar magnet is shown in Figure 19. If a piece of paper be placed over a bar magnet and iron filings sprinkled on the paper, they will arrange themselves, when the paper is gently tapped, into definite lines whose direction corresponds to the direction of the field. Magnetic lines are supposed to possess two properties, first, a tendency to shorten themselves and, second, they repel each other. The action between like and unlike poles can be accounted for by assuming the magnetic lines to possess the above properties.

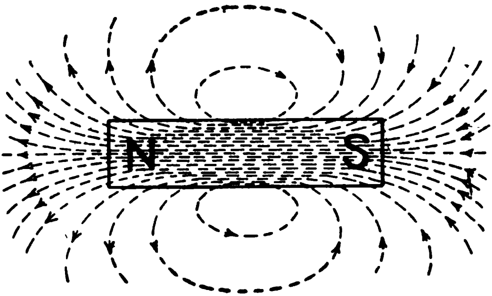


Figure 19.—Magnetic Field About a Bar Magnet.

*Distortion of Magnetic Field.*—The direction of a magnetic field is influenced by the presence of material having different magnetic properties. If a magnetic substance, such as a piece of soft iron, be introduced into a magnetic field that exists in air, the form of the field will be changed because the material is a better conductor of magnetic lines than air and, therefore, the lines crowd into the material. There will be a greater number of lines per unit area in the iron than there is per unit area in the air, the areas being taken perpendicular to the direction of the lines in each case. A distorted field is shown in Figure 20.

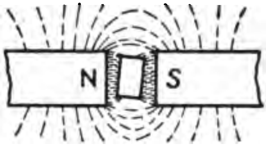


**Figure 20.—Distorted Magnetic Field.**

All materials that conduct magnetic lines better than air are called *paramagnetic,* and all materials that do not conduct as well as air are called *diamagnetic.*

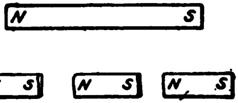


**Figure 21.—Poles Formed Where Magnet Is Broken.**

*Magnetic Screen.*—It is impossible to confine the magnetic lines forming a magnetic field to a definite path, as in the case of an electric current, for there is no insulator for magnetism as there is for the current. The influence of a magnetic field is oftentimes detrimental to the satisfactory operation of an electrical instrument, or other devices, and in such cases the effect of the field is reduced by surrounding the instrument or device with a housing of soft iron which conducts practically all of the lines that originally passed through the space occupied by the instrument or device. Such a housing is called a *magnetic shield,* or *screen.* In special cases several such screens are placed inside each other, each inner one serving to conduct the greater part of the lines not conducted by the screens outside of it.

*Applications of Permanent Magnets.*—Permanent magnets are very extensively used in the construction of electrical measuring instruments and other electrical devices. Two typical forms of permanent magnets are shown in Figure 22. The magnet at the left is similar to the one used in Weston direct-current instruments, and the one to the right is similar to the magnets used as damping magnets in watt-hour meters.

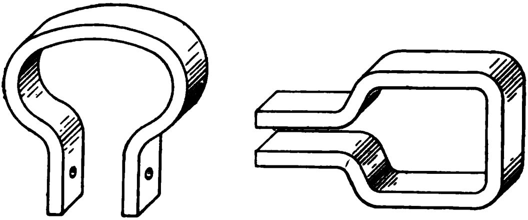


Figure 22.—Typical Forms of Permanent Magnets.

## ELECTROMAGNETISM

*Magnetic Field About a Conductor in Which There Is a Current.*—If a compass needle be brought near a horizontal conductor in which there is a current of electricity, it will be acted upon by a magnetic force which will tend to turn the needle so it will be at right angles to the axis of the conductor. The position occupied by the compass needle, when it comes to rest, will change with a change in the value of the current in the conductor and also with a change in the distance between the compass needle and the conductor. If the current in the conductor be reversed in direction and adjusted to exactly the same value as in the previous case, the deflection of the compass needle will be changed by approximately 180 degrees. The above experimentally determined facts prove: First, there is a magnetic field surrounding a conductor in which there is a current; second, the strength of the field produced by the current depends upon the value of the current; third, the strength of the field produced by the current varies with the distance from the conductor; and fourth, there is a definite relation between the direction of the magnetic field and the direction of the current in the conductor. Magnetism set up in this way by an electric current is called *electromagnetism.* Assuming the direction of current in the conductor corresponds to the arrow parallel to the conductor in Figure 23, then the north and the south poles of the compass needle will move in the direction of the curved arrows.

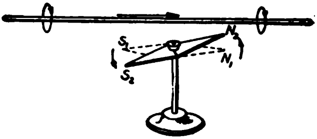


**Figure 23.—Magnetic Field About a Conductor Acting on a Compass Needle.**

*Direction of Magnetic Field About a Straight Conductor in Which There Is a Current.*—The direction of a magnetic field produced by a conductor is determined in exactly the same manner as the direction of a field due to a permanent magnet, namely, by noting the direction in which a north pole is urged when placed in the field. If a magnetic needle, which is suspended in such a manner that it is free to turn in all directions, be placed in a magnetic field, it will come to rest with the axis of the needle parallel to the direction of the field and the north pole will point in the direction of the field.

The direction of the magnetic field surrounding a conductor in which there is a current, as determined by a compass needle, is shown in Figure 24. If a conductor in which there is a current be passed through a sheet of paper and iron filings sprinkled on the paper, they will arrange themselves in more or less regular concentric circles, when the paper is gently jarred, as shown in Figure 25.

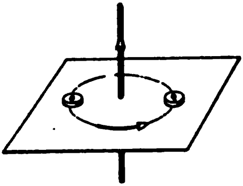


Figure 24.—Direction of the Magnetic Field About a Conductor in Which There Is a Circuit.

There are a number of simple methods of remembering the relation between the direction of a current in a conductor and the direction of the magnetic field due to the current. A very simple rule known as the "right-hand rule" is as follows: *Grasp the conductor with the right hand, the thumb being placed along the conductor and the fingers around the conductor; then the fingers will point in the direction of the magnetic field when the thumb points in the direction of the current in the conductor.*

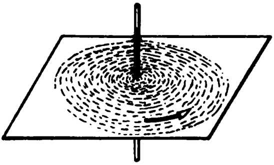


Figure 25.—Magnetic Field Formed from Iron Filings.

If a person looks along a conductor carrying a current in the direction of the current, the direction of the magnetic field surrounding the conductor will be clockwise. Two methods of representing the direction of a magnetic field in relation to the direction of the current are given in Figure 26. A cross-section of the conductor is shown in each case, and a current from the observer is indicated by a plus sign, while a current toward the observer is indicated by a minus sign. The field is indicated as being clockwise in one case and counter-clockwise in the other.

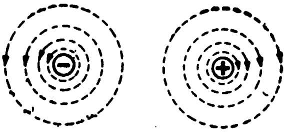


Figure 26.—Direction of Magnetic Field About a Conductor as Related to the Direction of the Current.

*Solenoid.*—If a conductor, carrying a current, be bent into a form similar to that shown in Figure 27 and a compass needle placed midway between the horizontal portions, it is apparent that the forces exerted on the needle, due to the current in the upper and the lower portions of the conductor, will be in the same direction. The effect of the two parts will be just double that produced by either part acting alone. Hence, if the conductor be coiled about the needle, each additional turn will produce an addi-

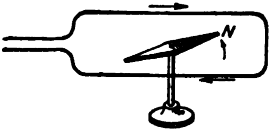


Figure 27.—Increasing the Magnetic Effect of the Current in a Conductor.

tional force tending to turn the needle from its normal position. The magnetic effect of any current can be greatly increased in this way.

A cross-section through a single turn of wire carrying a current and the magnetic field surrounding the turn are shown in Figure 28. The current is away from the observer in the upper part of the wire and toward the observer in the lower part of the wire, which results in the direction of the magnetic field about the upper part being clockwise and about the lower part counter-clockwise. It is apparent that the direction of the magnetic field between the upper and the lower cross-sections of the wire is toward the left,

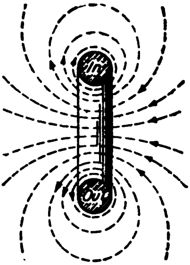


Figure 28.—Magnetic Field About a Coil of One Turn.

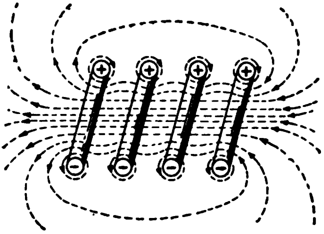


Figure 29.—Magnetic Field About a Coil of Several Turns.

and all the lines surrounding the wire pass through the coil in the same direction. The magnetic field is stronger in the center of the turn than it is outside, which is indicated by a larger number of lines of force per unit of area, as shown in the figure.

If the number of turns forming the coil be increased, the strength of the magnetic field inside the coil will be increased, since the lines of force that surround each turn seem to pass around the entire winding instead of passing around the individual turns. A cross-section through a coil composed of several turns is shown in Figure 29. A few of the lines encircle the individual turns, but the greater portion pass entirely through the center of the coil and around all the turns. Such coils are called *solenoids.*

*Polarity of Solenoids.*—A solenoid carrying a current exhibits all the magnetic effects that are shown by permanent magnets. They attract and repel magnets, pieces of iron and steel, other solenoids, etc. The magnetic lines pass through the solenoid from the south pole to the north pole and outside the solenoid from the north pole to the south pole, just as in a permanent magnet.

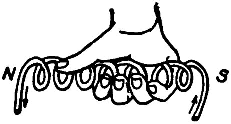


Figure 30.—How to Determine the Polarity of a Solenoid.

A simple method by which the polarity of a solenoid may be determined, if the direction of the current in the winding is known, is as follows: Grasp the solenoid with the right hand, placing the fingers around it in the direction of the current, the thumb will then point in the direction of the north pole, as shown in Figure 30.

The action of a solenoid on a compass needle is shown in Figure 31.

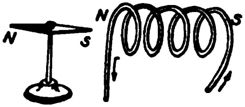


Figure 31.—Action of a Solenoid on a Compass Needle.

*Magnetomotive Force.*—When a current of electricity is produced in the winding of a solenoid, it becomes magnetized and lines of force pass through its interior from the south to the north pole and return outside from the north to the south pole. The current sets up a force which drives the lines of force, called *magnetic flux,* through the path which they take, called the *magnetic circuit,* just as the electromotive force in the electrical circuit causes the electricity to flow through the electrical circuit. This force, due to the current, is called the *magnetomotive* force, m.m.f. (abbreviated). The magnetomotive force of a solenoid is directly proportional to the product of the number of turns in the solenoid and the current, in amperes, the turns are carrying. If $N$ represents the number of turns in the solenoid and $I$ the current in amperes passing through the winding, the magnetomotive force will vary directly as $NI$. The product of the current and the turns is known as the *ampere-turns,* and the same magnetomotive force is obtained with a current of 500 amperes through 1 turn, 25 amperes through 20 turns, or 1 ampere through 500 turns. In each of the above cases the magnetomotive force is 500 ampere-turns.

The unit for magnetomotive force usually used in magnetic calculations is the *gilbert.* To change from ampere-turns to gilberts, it is necessary to multiply by $(4\pi \div 10)$ or 1.2564.

*Reluctance.*—The magnetomotive force acting on any magnetic circuit encounters a certain opposition to the production of a magnetic flux, just as the electrical pressure encounters a certain opposition in the electrical circuit to the production of a current. The opposition offered by the magnetic circuit is called its *reluctance,* and it is represented by the symbol $S$. The reluctance of a magnetic circuit depends upon the material composing the magnetic circuit and upon the dimensions of the circuit. It varies directly as the length of the circuit, inversely as the area, all other conditions remaining constant, and inversely as a property of the material called its permeability, which will be defined later, represented by the symbol $\mu$.

$$\text{reluctance} = \frac{\text{length of circuit in centimeters}}{\text{permeability} \times \text{cross-section in square centimeters}}$$

$$S = \frac{l}{\mu A}$$

The unit in which reluctance is measured is called the *oersted* and it is equal to the reluctance of a cubic centimeter of air.

Reluctances are added in the same way as resistances. If the magnetic circuit is composed of several different materials, the reluctance of each part may be computed separately and the results added, which will give the reluctance of the entire circuit.

*Ohm's Law for the Magnetic Circuit.*—The magnetomotive force acting on a magnetic circuit, the reluctance of the circuit, and the magnetic flux produced are related to each other just as the electrical pressure in an electrical circuit, its resistance, and the current produced are related to each other. The unit of magnetic flux is the *maxwell* and it corresponds to one line of force. The symbol for magnetic flux is $\phi$.

$$\text{magnetic flux} = \frac{\text{magnetomotive force}}{\text{reluctance}}$$

$$\text{maxwells} = \frac{\text{gilberts}}{\text{oersteds}}$$

$$\phi = \frac{\text{m. m. f.}}{S}$$

$$\phi = \frac{1.2564\,NI}{\dfrac{l}{\mu A}}$$

$$= \frac{1.2564\,NI\mu A}{l}$$

*Intensity of Magnetic Field and Induction Density.*—The strength or intensity of a magnetic field produced by a current at any point is numerically equal to the force in dynes which would act on a unit magnetic pole if it were placed at the point. Field strength for a uniform field in air is equal to the total flux divided by the area of the field perpendicular to the lines. It is also equal to the magnetomotive force in gilberts acting on the magnetic circuit divided by the length of the circuit in centimeters. Field strength is represented by the symbol $H$.

$$H = \frac{\phi}{A} \qquad \text{(a)}$$

also

$$H = \frac{\text{m. m. f.}}{l} \qquad \text{(b)}$$

$$= \frac{1.2564\,NI}{l}$$

The unit in which field strength is usually measured is the *gauss*, and it is equal to one line of force per square centimeter. Equation (a) gives the field strength in gausses. Field strength is frequently expressed as so many gilberts per centimeter, as given in equation (b).

If the magnetomotive force of a solenoid with an air core is maintained constant and an iron core is introduced, the magnetic flux per unit area will be greatly increased. The permeability of a medium is equal to the induction density divided by the field intensity to which the induction is due, and it is represented by the symbol $\mu$. The magnetic lines per unit area in the iron is called *flux density* or *induction density,* and it is represented by the symbol $B$.

$$\mu = \frac{B}{H}$$

*Example.*—A magnetic circuit 50 centimeters in length and 10 square centimeters in area is composed of a material having a permeability of 100. Determine the magnetic flux produced by a current of 5 amperes through a winding of 1,000 turns about the circuit.

*Solution.*—Substituting in the equation given in the previous section, which gives the value of the magnetic flux in maxwells in terms of magnetomotive force in gilberts and the reluctance in œrsteds, we have

$$\phi = \frac{1.2564 \times 1{,}000 \times 5 \times 100 \times 10}{50} = 125{,}640 \text{ maxwells}$$

*Magnetization Curves.*—The permeability of iron is not constant but depends upon the degree to which it is magnetized, hence, in order to compute the reluctance of a magnetic circuit, it is necessary to know either the field intensity or the induction density, and its relation to the permeability. A magnetization curve is a curve showing the relation between the magnetizing force and the induction density for a given grade of material. The magnetizing force is usually plotted on the horizontal and the induction density on the vertical. The relation between the induction density and the magnetizing force is different during the increase of magnetizing force from what it is during the decrease of magnetizing force. The average relation between $B$ and $H$ for several samples of iron are shown in Figure 32. The tendency for the induction density to lag behind the magnetizing force is called *hysteresis,* and it is the source of considerable trouble in the operation of certain instruments. Figure 33 shows the form of curve obtained when a piece of iron is first magnetized in one direction to a certain induction density, then the magnetizing force is reduced to zero and built up to an

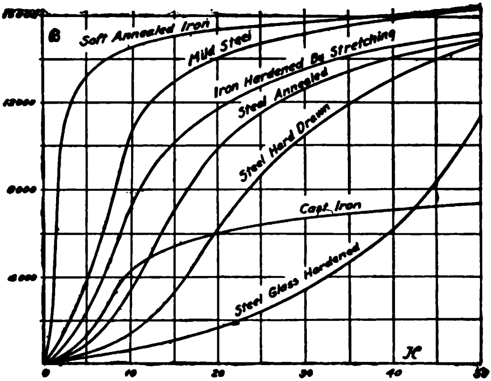


Figure 32.—Magnetization Curves.

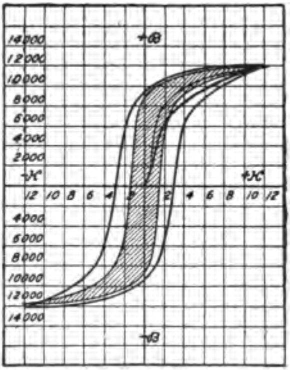


Figure 33.—B-H Curves.

equal value in the reverse direction, then reduced to zero and built up to its original maximum value. Such a curve is called a *B-H* curve.

The value of the induction density, when the field intensity is zero, is called the *remnant,* or *residual magnetism,* of the iron, and the value of the field intensity when the induction density is zero is called the *coercive force.*

*Retentiveness.*—An examination of the *B-H* curves given in Figure 33 will make the fact clear that certain materials are better suited for making permanent magnets than others. The two curves are for materials which retain about the same residual magnetism, but one will lose its magnetism a great deal easier than the other, due to rough handling, etc., as its coercive force is so much smaller. The property of a material for retaining its magnetism is called its *retentiveness.*

*Hysteresis Loss.*—When a sample of iron is carried through what is called a *magnetic cycle,* all of the energy spent in magnetizing it is not returned to the circuit when the iron is demagnetized, which results in a certain amount of electrical energy being expended to carry the iron through the cycle. This energy will appear as heat, and the amount expended per cycle depends upon the kind of iron being tested, the volume of the sample, and the maximum value of the induction density *B* raised to the 1.6 power.

$$\text{joules (energy per cycle)} = V \times B^{1.6} \times K \times 10^{-7}$$

in which *V* represents the volume of the sample in cubic centimeters, and *K* is a constant taking into account the kind of iron being tested, which is called the *hysteretic constant* of the material.

When the sample is carried through *f* magnetic cycles per second, the loss of power in watts is given by the following equation:

$$P_h = V \times K \times B^{1.6} \times f \times 10^{-7} \text{ watts}$$

### TABLE I

### VALUE OF HYSTERETIC CONSTANT K FOR DIFFERENT MATERIALS

| | |
|---|---|
| Ordinary Sheet Iron | .004 |
| Thin Sheet Iron (good) | .003 |
| Best Annealed Transformer Sheet Metal | .0015 |
| Cast Steel | .012 |
| Cast Iron | .016 |
| Forged Steel | .025 |

## ELECTROMAGNETIC INDUCTION

*Electromotive Force Induced in a Conductor.*—If a conductor be moved in a magnetic field in such a manner that it cuts some or all of the magnet lines forming the field, there will be an electromotive force induced in the conductor. If this conductor forms part of a closed electrical circuit, the electromotive force induced in it will produce a current. Currents produced in this way are called *induced currents* and the phenomenon is termed *electromagnetic induction.*

*Direction of Induced Electromotive Force in a Straight Conductor.*—There is a definite relation between the direction of motion of a conductor, the direction of the magnetic field, and the direction of the induced electromotive force, which may be proved by a simple experiment similar to the one shown in Figure 34. A conductor *C* is connected in series with

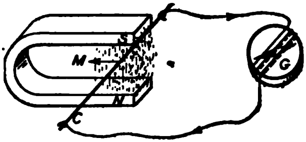


Figure 34.—Moving a Conductor in a Magnetic Field.

a small galvanometer *G*, and a permanent magnet provides a magnetic field in which the conductor may be moved. If the conductor be moved from right to left across the field, the needle of the galvanometer will be deflected. When the conductor is moved from left to right across the field, the galvanometer will be deflected but in the opposite direction to what it was in the first case. If the conductor be moved directly across from one pole of the magnet to the other or parallel to the magnetic field there will be no deflection of the galvanometer, which indicates there is no induced electromotive force in the conductor. The same results can be obtained by moving the magnet.

The direction of the electromotive force induced in the conductor will always be such that the current produced by it, if any, will increase the strength of the magnetic field on the side of the conductor in the direction it is moving and decrease it on the opposite side. For example, in Figure 34 when the conductor is moving toward the left, the current in the conductor *C* will pass from the front end toward the back, which will result in the magnetic field being weakened on the right-hand side and strengthened on the left-hand side. A cross-section through a conductor and the magnetic field about it are shown in Figure 35. A simple rule, known as the "right-hand"

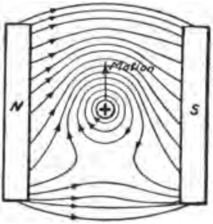


Figure 35.—Field About a Conductor, in Which There Is a Current, When It Is Moving in a Magnetic Field.

rule, for determining the relation between the direction of motion, the direction of the magnetic field, and the direction of the induced electromotive force is as follows: *Place the thumb, forefinger, and middle finger of the right hand at right angles to each other,* as shown in Figure 36, *then the middle finger will point in the direction of the induced electromotive force when the thumb points in the direction of motion and the forefinger in the direction of the magnetic field.*

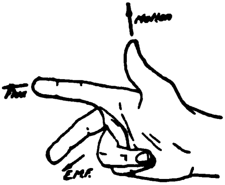


**Figure 36.—Right-Hand Rule for Determining Induced Electromotive Force.**

If a current be sent through the conductor from an outside source, it will tend to move in the opposite direction to that indicated in Figure 36. If the left hand be used in this case instead of the right, the direction in which the conductor moves or tends to move will correspond to the direction in which the thumb points.

*Direction of Induced Electromotive Force in a Coil.*—If a coil of wire be connected in series with a galvanometer and a magnet thrust into the coil, there will be an electromotive force induced in the coil which will cause a current to flow through the circuit and deflect the moving element of the galvanometer. If the magnet be withdrawn from the coil, the galvanometer will be deflected in the opposite direction to what it was when the magnet was thrust into the coil, because the relative motion of the magnetic field and the conductor, or turns in the coil, has been reversed. The direction of the induced electromotive force will always be such that the current produced by it, if any, will flow around the coil in such a direction that its magnetic effect tends to prevent a change in the number of magnetic lines through the coil. For example, if the magnet is moving into the coil, as shown in Figure 37, the current in the coil, due to the induced electromotive force, will be in such a direction as to make the left-hand end of the coil a south pole and the right-hand end a north pole, or the magnetic effect of the current tends to prevent the flux changing, or, in this case, building up through the coil. If, on the other hand, the magnet is moving out of the coil, as shown in Figure 38, the current in the coil, due to the induced electromotive force, will be in such a direction as to make the left-hand end of the coil a north pole and the right-hand end a south pole, or the magnetic effect of the current tends to prevent the flux changing, or, in this case, decreasing in value. The direction of the current due to the induced electromotive force is indicated by arrows in both cases, as shown in Figures 37 and 38. It must be remembered that this tendency to prevent a change in flux through the coil is present only when the coil and the magnet are moving with respect to each other, for when there is no such movement there is no induced electromotive force and, hence, no current.

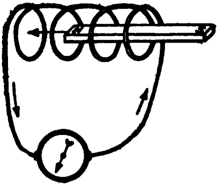


**Figure 37.—Magnet Moving Into Coil.**

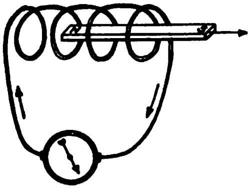


**Figure 38.—Magnet Moving Out of Coil.**

*Electromotive Force Induced in a Circuit Due to a Change of Current in the Circuit Itself or Another Circuit.*—If the current in a conductor changes in value, there will be a change in the magnetic field surrounding the conductor and, hence, there will be a relative motion of the conductor with respect to the field, which will result in an induced electromotive force being produced in the conductor. The direction of this induced electromotive force will always be in such a direction as to tend to prevent the current changing in value; that is, if the current is increasing in value, the induced electromotive force acts against it and if the current is decreasing in value the induced electromotive force acts with it or in the same direction. As a result of this induced electromotive force being produced by a change of current in a circuit, the current cannot build up to its maximum value or decrease to zero value instantly. The property of a circuit, which results in an induced electromotive force being produced in the circuit due to a change of current in the circuit itself, is called the *self-inductance* of the circuit.

If two circuits, which are electrically independent, are so placed with respect to each other that all or a part of the magnetic lines produced by the current in one circuit pass through the other circuit, or *vice versa,* an electromotive force will be produced in one due to a change of current in the other, since there is a relative movement of the magnetic field and the con-

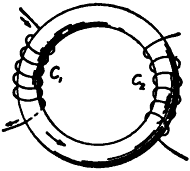


Figure 39.—Two Coils About a Common Magnetic Circuit.

ductor. Two coils placed about a common magnetic circuit, as shown in Figure 39, act in the above manner with respect to each other. When the current in one coil changes, there will be a current in the other coil, if its circuit is closed, as a result of the induced electromotive force and the direction of this current will always be such as to tend to prevent a change in the value of the magnetic flux through the two coils. If the current in one coil increases, the current in the other is in the opposite direction around the magnetic circuit; while if the current decreases in one coil, the current in the other is in the same direction around the magnetic circuit and tends to maintain the magnetic flux constant in value. The property of a circuit, which results in an electromotive force being induced in it due to a change of current in another independent circuit, is called the *mutual inductance* of the circuit with respect to some other circuit.

*Factor upon Which the Induced Electromotive Depends.*—The value of the induced electromotive force in a conductor depends upon the following factors:

*First,* the rate of movement of the conductor and the magnetic field with respect to each other. The more rapid the movement the greater the induced electromotive force, all other quantities remaining constant.

*Second,* the strength of the magnetic field, or the number of lines of force per unit of area perpendicular to the direction of the field. The stronger the field the greater the induced electromotive force, all other quantities remaining constant.

*Third,* the angle the path in which the conductor moves makes with the direction of the magnetic field. The nearer this path is to being perpendicular to both the direction of the magnetic field and the axis of the conductor, the greater the induced electromotive force.

*Fourth,* the length of the conductor that is actually in the magnetic field. The more wire there is in the field, the greater the induced electromotive force, all other quantities remaining constant.

The above facts can be condensed into the following simple statement: The value of the induced electromotive force in any circuit depends upon the resultant rate at which the conductor forming the circuit cuts the magnetic lines; that is, it depends upon the resultant number of magnetic lines cut by the conductor in a unit of time. If a loop of wire be moved in a magnetic field in such a manner that both sides are cutting flux in the same direction and at the same rate, the resultant change in flux enclosed by the loop will be zero and, hence, the resultant electromotive force will also be zero. There will be an electromotive force induced in each side of the coil, but they will act in opposite directions around the loop. There must be a change in the magnetic flux enclosed by a loop in order that there be a resultant electromotive force induced due to a movement of the loop and magnetic field. When the motion of the conductor and the magnetic field with respect to each other results in one hundred million (100,000,000) lines being cut in each second, there is an electromotive force of one volt induced in the conductor.

*Examples.*—1. A conductor 100 centimeters in length is moved at a velocity of 50 cm. per second perpendicular to itself and the direction of a uniform magnetic field having an intensity of 10,000 gausses. What is the value of the induced electromotive force in volts?

*Solution.*—The total magnetic flux cut by the conductor in each second will be equal to the product of the length of the conductor in centimeters, the velocity of the conductor in centimeters per second, and the strength of the magnetic field in lines per square centimeter, or gausses.

$$\begin{aligned}\phi &= l \times v \times H \\ &= 100 \times 50 \times 10{,}000 \\ &= 50{,}000{,}000 \text{ maxwells}\end{aligned}$$

Since the conductor must cut 100,000,000 lines per second in order to have an induced electromotive force of one volt, the induced electromotive force in this case will be

$$\begin{aligned}E &= 50{,}000{,}000 \div 100{,}000{,}000 \\ &= .5 \text{ volt}\end{aligned}$$

2. The magnetic flux through a coil of 1,500 turns is changing at the rate of 500,000 lines per second, what is the value of the induced electromotive force?

*Solution.*—Each of the 1,500 turns will cut the change in magnetic flux, or the result is equivalent to one turn cutting a change in magnetic flux 1,500 times as great.

$$\begin{aligned}\text{Flux cut per second} &= 1{,}500 \times 500{,}000 \\ &= 750{,}000{,}000 \text{ maxwells} \\ E &= 750{,}000{,}000 \div 100{,}000{,}000 \\ &= 7.5 \text{ volts}\end{aligned}$$

*Eddy Currents.*—When a mass of metal is moved in a magnetic field or the magnetizing force acting on a magnetic circuit continuously changes, currents, called *eddy currents,* will flow through the metal. These currents heat the metal and represent a loss in power in the operation of dynamo-electric machinery. If a metal disk be revolved between the poles of a permanent magnet, as shown in Figure 40, eddy currents will be produced in the disk which will tend to oppose its free rotation. The relative direction of these currents, the direction of the field, and the direc-

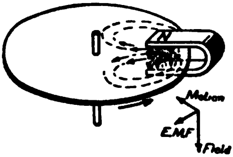


Figure 40.—Revolving a Metal Disk in a Magnetic Field.

tion of the motion are shown in the figure. The currents circulate in local paths or eddies and for this reason are called eddy currents.

The loss of power due to eddy currents is greatly reduced in electrical apparatus by constructing the volume in which they occur of thin sheets of metal, called *laminations,* arranged in such a way that their planes are perpendicular to the direction in which the eddy currents tend to flow. A good example of such construction is found in the cores of transformers and armature cores.

A good example of the practical application of eddy currents is found in almost all types of integrating meters. In these cases there is a metal disk mounted on the axis of the revolving portion of the meter and so arranged that it revolves between the poles of one or more permanent magnets. The torque required to drive such a disk is practically directly proportional to the speed, which is a very desirable relation in this particular case, as will be pointed out later.

*Calculation of Eddy-Current Loss.*—The eddy-current loss in watts occurring in a given volume of iron may be computed by means of the following equation:

$$P_e = V \times f^2 \times t^2 \times B^2 \times K$$

in which $V$ is the volume of the iron in cubic centimeters; $f$ is the frequency of the magnetic cycles per second; $t$ is the thickness of the laminations in centimeters; $B$ is the maximum induction density in lines per square centimeter; and $K$ is a constant, depending upon the resistance of the iron per cubic centimeter, which is usually about $1.6 \times 10^{-11}$.

# CHAPTER III

## INDUCTANCE AND ITS MEASUREMENT

*Action of Magnetic Field about a Conductor Carrying a Variable Current.*—There is a magnetic field about every conductor in which there is a current, and the direction and the strength of this field will depend upon the direction and the value of the current in the conductor. The relation between the direction of the field surrounding a conductor due to a current in the conductor and the direction of the current in the conductor is shown in Figure 41.

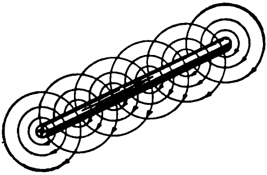


Figure 41.—Magnetic Field Surrounding a Conductor in Which There Is a Current.

With any change in the value of the current in the conductor, there will be a change in the number of magnetic lines surrounding the conductor, their number increasing with an increase in current and decreasing with a decrease in current. These magnetic lines are supposed to originate at the center of the conductor and to increase in diameter with an increase in current, the first lines formed being crowded farther away from the conductor by each new line; and to decrease in diameter with a decrease in current, the lines nearest the center of the conductor disappearing.

*Self-Induction.*—As a result of the movement of the magnetic lines with respect to the conductor, as described in the previous section, there will be an electromotive force induced in the conductor. The direction of this induced electromotive force will depend upon the relation between the movement of the field and the conductor with respect to each other. With an increasing current, the induced electromotive force will be opposite that produced by a decreasing current. A cross-section of a conductor carrying a current from the observer is shown in Figure 42. Let us take a small section of the conductor, such as *A* on the right-hand side, and imagine the current is decreasing in value, which would be equivalent to the section of the conductor moving toward the right through the field. Now, applying the right-hand rule for determining the direction of the induced electromotive force, you will find it is from you, or in the same direction as the current in the conductor. With an increasing current, the result will be just the reverse. It is apparent that the action of this induced electromotive force is always such as to tend to prevent a change in the value of the current in the conductor.

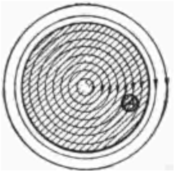


**Figure 42.—Movement of a Magnetic Field Across a Conductor.**

There will be no electromotive force induced in the conductor unless there is a change in the value of the current, for the value of the current must change in order that there be a relative movement of the field and conductor. The value of the induced electromotive force will depend upon the number of magnetic lines cut by the conductor in a given time.

The property of a circuit, which results in an electromotive force being induced in it due to a change in the value of the current it is carrying, is called its *self-inductance,* and the process is called *self-induction.*

*Hydraulic Analogy of Self-Inductance.*—The self-inductance of an electrical circuit is, in action, very similar to the inertia of the water in the hydraulic circuit. The inertia of the water tends to prevent a change in its velocity, it acting in the same direction as the water flows when the rate of flow is decreasing and in the opposite direction when the rate of flow is increasing.

A part of an hydraulic circuit is shown in Figure 43, which depicts a small water motor driving a large fly wheel. The action of this combination in the hydraulic circuit, when the current of water changes in value, is very similar to the action of self-inductance in an electrical circuit when the current of electricity changes in value. The inertia of the heavy fly wheel prevents the water motor speeding up as fast as it would if it were operating alone, and it also prevents the speed of the motor decreasing as rapidly as it would if it were not connected to the fly wheel.

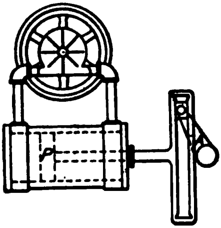


**Figure 43.—Hydraulic Analogy of Inductance.**

*Unit of Self-Inductance.*—The unit of self-inductance is defined in terms of the induced electromotive force produced by a given change of current per unit of time, since the total number of magnetic lines surrounding the conductor varies with the current. If the current in a circuit changes in value at a uniform rate of one ampere per second and, as a result, there is an electromotive force of one volt induced in the circuit, the circuit is said to have one unit, or one henry, of self-inductance. Any change in the form of a circuit or the material surrounding a circuit, which will result in a greater electromotive force being induced in the circuit due to a certain uniform change of current in a given time, will increase its self-inductance. Likewise, any change in the form of a circuit or the material associated with it, which will result in a decrease in the induced electromotive force due to a given uniform change of current in a given time, will decrease the self-inductance of the circuit. Self-inductance is represented by the letter $L$.

*Self-Inductance of a Long Solenoid.*—In calculating the inductance of a circuit, it is necessary first to determine the change in magnetic flux produced by a change in current of one ampere, then multiply this change in flux by the number of times the conductor forming the circuit cuts the flux, and then divide this result by 100,000,000, or $10^8$, to reduce it to volts.

The total magnetic flux passing through a solenoid of $N$ turns, with an iron core $A$ square centimeters in cross-section and $l$ centimeters in length, and having a permeability of $\mu$ due to a current of one ampere, is given by the following equation:

$$\phi = \frac{4\pi \times N \times \mu \times A}{10l}.$$

Multiplying this change in flux, due to a change in current of one ampere, by the number of turns in the coil gives the total flux cut by the conductor forming the circuit. If the current changes at the rate of one ampere per second, or if this flux is cut in one second, the induced electromotive force will be equal to the total flux cut divided by 100,000,000. Since, by definition, the inductance of a circuit is equal to the induced electromotive force due to a change in current of one ampere per second, we may write

$$L = \frac{4\pi \times N \times N \times \mu \times A}{10l \times 100,000,000}$$

$$= \frac{4 \times N^2 \times \mu \times A}{10^9 l} \text{ henries}$$

*Example.*—What is the self-inductance of a solenoid composed of 1,500 turns, about an iron core 50 centimeters long, 25 square centimeters in area, and having a permeability of 1,000?

*Solution.*—Substituting in the above equation gives,

$$L = \frac{4 \times 3.1416 \times 1,500 \times 1,500 \times 1,000 \times 25}{1,000,000,000 \times 50} = 14.14\text{—henries}$$

*Inductance of Electrical Circuits.*—The accurate calculation of the inductance of electrical circuits, in general, is quite complicated, but the reader may find the final, or working, form of the equations in textbooks on the subject and in the different handbooks for electrical engineers.

The self-inductance in millihenries per 1,000 feet of each of two parallel non-magnetic wires, $d$ units in diameter and $D$ units apart, may be computed by the use of the following equation:

$$L = 0.01524 + 0.1403 \log_{10} \frac{2D}{d}$$

The values of $D$ and $d$ in the above equation must be expressed in the same units.

*Energy Stored in Magnetic Field.*—A certain amount of energy must be expended in producing a current in a circuit containing inductance, just as energy must be expended in the hydraulic circuit, shown in Figure 43, in setting the fly wheel in motion. The energy expended in overcoming the inductance is stored in the magnetic field surrounding the circuit, while the energy in the hydraulic circuit is stored in the fly wheel. The energy stored in the magnetic field will depend upon the inductance of the circuit and the current through the circuit. Any change in the current will result in a change in the energy associated with the circuit, the direction of the transfer depending upon whether the current is increasing or decreasing in value. The energy in joules associated with the circuit may be computed by means of the following equation:

$$W = \tfrac{1}{2} L I^2 \text{ joules}$$

The following simple experiment will serve to prove that there is some energy associated with an inductive

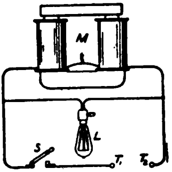


Figure 44.—Experiment Illustrating Effect of Inductance.

circuit, or one containing inductance. If an incandescent lamp *L* be connected across the terminals of an electromagnet *M*, as shown in Figure 44, the lamp will flare up in brightness when the switch *S* is opened due to the discharge of energy from the magnetic field. The working voltage of the lamp should be approximately equal to the pressure between the terminals $T_1$ and $T_2$ of the magnet for the best results.

Resistances are frequently provided for the field of large generators to discharge through when they are disconnected from the source of power, as the pressure generated in the winding of the field, without a resistance, may be sufficient to puncture the insulation.

*Mutual Inductance.*—If two conductors be placed parallel to each other and a current produced in one, a part of the magnetic field produced by this current will move across the second conductor and, as a result, there will be an electromotive force induced in it. When two conductors or circuits are so related that a change of current in one results in an electromotive force being induced in the other, they are said to have a mutual inductance and the process is called *mutual induction.* The unit of mutual inductance is the same as that for self-inductance, or the henry. One circuit is said to have a mutual inductance of one henry with respect to another when a uniform change of current of one ampere per second in one will produce an induced electromotive force of one volt in the other. Induction coils, transformers, etc., depend upon this principle of mutual induction for their action.

*Mutual Induction of Two Solenoids.*—The equation for the value of the mutual inductance of two long solenoids wound upon the same core is very similar in form to that for the value of the self-inductance of a long solenoid. In this case the flux produced by a change in current in one coil is cut by the turns in the other coil. Representing the turns in the two coils by $N_1$ and $N_2$, the length of the solenoids in centimeters by $l$, the area of the core in square centimeters by $A$, and the permeability of the material forming the cove by $\mu$, we may write

$$M = \frac{4\pi \times N_1 \times N_2 \times \mu \times A}{10^9 \times l} \text{henries}$$

*Example.*—Calculate the mutual inductance between two solenoids of 500 and 1,000 turns, respectively, wound on an iron core 20 centimeters long, 20 square centimeters in area, and having a permeability of 1,000.

*Solution.*—Substituting in the above equation for mutual inductance gives

$$M = \frac{4 \times 3.1416 \times 500 \times 1{,}000 \times 1{,}000 \times 20}{1{,}000{,}000{,}000 \times 20} = 6.2832 \text{ henries}$$

*Bridge Method of Measuring Self-Inductance.*—The value of an unknown inductance may be determined in terms of a known inductance by means of a Wheatstone net, as shown diagrammatically in Figure 45. In this figure $L_s$ is the known inductance, and $L_x$ the inductance whose value is to be determined. The resistances marked $A$ and $B$ are the ratio arms of the bridge, and $R_1$ and $R_2$ are auxiliary resistances for the purpose of adjusting the balance of the bridge. The two right-hand arms of the bridge should be adjusted so that they are equal in value, say 1,000 ohms each, and the ratio of the two left-hand arms varied until the galvanometer shows no deflection when its circuit is closed and the battery key tapped. After this adjustment is obtained

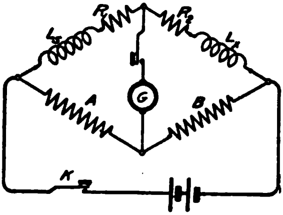


Figure 45.—Bridge Method of Measuring Inductance.

$$L_x = L_s \frac{B}{A}$$

The values of the resistances $R_1$ and $R_2$ are supposed to include the resistance of the two inductances $L_s$ and $L_x$, respectively.

A balance may be readily obtained by substituting a source of alternating current for the battery and a telephone receiver for the galvanometer and adjusting the ratio resistances until there is a minimum sound in the receiver.

*Bridge Method of Measuring Mutual Inductance.*—The mutual inductance between two coils having self-inductances of $L_1$ and $L_2$ henries may be determined by first measuring the effective inductance of the two coils in series and their fields in the same direction and then measuring the effective inductance with them in series and their fields opposed. These measurements may be made as in the previous section. Calling the effective inductance $L_c$, when the fields of the coils are in the same direction, and $L_o$ when the fields are in the opposite direction, let $M$ represent the mutual inductance between the two coils. Then we have

$$L_c = L_1 + 2M + L_2$$
$$L_o = L_1 - 2M + L_2$$

*Voltmeter Ammeter Method of Measuring Inductance.*—The drop in pressure over an inductive circuit in which there is an alternating current is composed of two parts, which are displaced in phase by 90 degrees. The drop over the resistance is equal to $RI$, and the drop over the inductance is equal to $2\pi f L I$, in which $R$ is the resistance of the circuit in ohms, $I$ is the effective current in amperes, $f$ is the frequency of the current, and $L$ is the inductance of the circuit in henries. The total drop over the circuit, as indicated by a voltmeter, is equal to the vector sum of the drops as calculated from the above expressions. Since these drops or components are at right angles to each other, the resultant is equal to the square root of the sum of the squares of the components, or

$$E = \sqrt{(RI)^2 + (2\pi fLI)^2}$$

All of the quantities in the above equation with the exception of $L$ may be readily determined; and when their values are substituted in the equation, the value of $L$ may be computed.

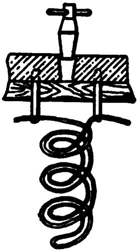


Figure 46.—Non-Inductive Coil.

*Non-Inductive Coil.*—A non-inductive coil is one wound in such a manner that the magnetic effect of a portion of the turns is neutralized by the magnetic effect of the remaining turns. Such a coil is shown in Figure 46.

# CHAPTER IV

## CAPACITY AND ITS MEASUREMENT

*Condenser.*—Two conductors separated by some insulator, such as air, rubber, glass, mica, or paraffined paper, constitute what is called a *condenser,* and it is said to have electrical capacity. The medium separating the two conductors is called the *dielectric.* If the two conductors forming the condenser be connected to the terminals of a battery or other source of electrical pressure, as shown in Figure 47, they will acquire equal and opposite charges of electricity, or the condensor becomes charged. The sign of the charge on each conductor will correspond to the sign of the terminal of the source of electrical pressure to which it is connected. The final difference in electrical pressure between the conductors forming the condenser will be equal to the value of the impressed electrical pressure.

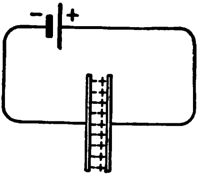


**Figure 47.—Condenser Connected to a Battery.**

*Charging a Condenser.*—The action of a condenser in an electrical circuit is very similar to the action of an elastic diaphragm stretched across an hydraulic circuit. The action of such a diaphragm can be easily understood by referring to Figure 48. When the diaphragm is in a vertical position, it offers no resistance to the flow of water through the pipe; but, just as soon as it is deflected from this position, the slightest amount to the right or to the left, it immediately tends to prevent the water pressing through the pipe. This tendency of the diaphragm to prevent the water flowing through the pipe will continue to increase as it is stretched from the normal, or vertical position, until the opposition offered by the diaphragm is equal to the pressure acting on the circuit, which will result in zero current in the circuit. If the pressure acting on the circuit be changed, there will be a flow of water in the circuit in such a direction as to increase or decrease the strain in the diaphragm, depending upon whether the pressure is increased or decreased. The diaphragm may be held in its strained, or extended, position if the pressure acting on the circuit be disconnected by closing a valve on each side of the chamber in which the diaphragm is placed. The strained diaphragm may be made to do useful work by connecting a water motor or pump to the chamber in which it is placed and then opening the valves.

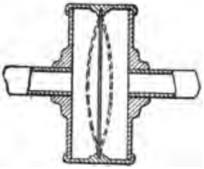


Figure 48.—Hydraulic Analogy of the Condenser.

A similar action is supposed to take place in the electrical circuit when a condenser is charged. There is a strain set up in the dielectric due to the electricity passing through it; and, when the condenser is fully charged, this strain is equal to the impressed electrical pressure. Any change in impressed electrical pressure will result in a flow of electricity through the dielectric in such a direction as will bring about the required change in strain in the dielectric. If a condenser be connected to a source of electrical pressure and then disconnected, there will be a certain quantity of energy stored in it; just as in the case of the diaphragm in the hydraulic circuit, and this energy may be used by allowing the condenser to discharge itself through an electrical circuit.

*Capacity of a Condenser.*—A condenser is said to have unit capacity when a transfer of unit quantity or electricity, or one coulomb, through the dielectric will produce a change of one volt in the difference in electrical pressure between its terminals, or the conductors forming the condenser. The unit of capacity, as defined above, is called the *farad.*

The capacity of any condenser will depend upon the area of the conductor, or plates, as they are usually called; the number of plates; the distance between the plates, or the thickness of the dielectric; and the kind of material composing the dielectric. The capacity of a condenser increases with an increase in the area of plates or with an increase in the number of plates, which is equivalent to an increase in area, and decreases with an increase in the distance between the plates.

The value of an insulating material as a dielectric is called its *specific inductive capacity.* The inductive capacity of different materials varies considerably, but it is less for air than it is for any solids or liquids. The specific inductive capacity of a material is measured in terms of air as a standard, and it is the ratio of the capacities of two condensers, one of which has the material, whose dielectric constant is to be determined, as the dielectric, and the other has air as the dielectric. The dimensions of the condensers must be the same in both cases. Table II gives the value of the specific inductive capacity of some of the materials that are commonly used in the construction of condensers and insulating conductors.

TABLE II

SPECIFIC INDUCTIVE CAPACITIES

| Material | Specific Inductive Capacity | Material | Specific Inductive Capacity |
|---|---|---|---|
| Air | 1.00 | Porcelain | 4.38 |
| Alcohol (Amyl) | 15.50 | Resin | 2.48 to 2.57 |
| Glass (Plate) | 3.00 to 7.00 | Rubber (Vulcanized) | 2.7 |
| Gutta-Percha | 3 to 5 | Shellac | 2.95 to 3.1 |
| Mica | 4.5 to 7 | Sulphur | 2.24 to 3.90 |
| Paraffine | 1.90 to 2.40 | Turpentine | 2.15 to 2.28 |
| Petroleum | 2.02 to 2.2 | Vaseline | 2.20 |

*Relation of Impressed Electrical Pressure, Quantity of Electricity, and Capacity of Condenser.*—If a condenser is said to have a unit of capacity when 1 coulomb of electricity will produce a change in the difference in electrical pressure between its terminals of 1 volt, then 1 volt change in impressed pressure between its terminals will produce a change of one coulomb in the value of its charge. If the capacity of a condenser is constant, the quantity of charge is directly proportional to the impressed electrical pressure; or, if the impressed pressure is constant, the quantity of charge is directly proportional to the capacity of the condenser. These two relations may be expressed in the form of an equation as follows:

$$Q = CE$$

in which $Q$ stands for quantity in coulombs, $C$ for the capacity of the condenser in farads and $E$ for the impressed pressure in volts. The farad is a very large unit of capacity, and a unit called the *microfarad,* which is equal to the one-millionth part of a farad, is more commonly used. The common values of capacities as used in the majority of cases are only a few microfarads.

*Capacity of Plate Condenser.*—The capacity of a simple condenser composed of two plain parallel plates separated by a dielectric may be calculated by means of the following equation:

$$C=\frac{K\times A}{4.452\times 10^{12}\times d}$$

in which $C$ is the capacity in farads; $K$ is the value of the dielectric constant of the dielectric; $A$ is the area of the plates, in square inches, which are exposed to each other; and $d$ is the distance in inches between the plates.

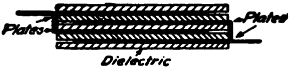


Figure 49.—Condenser of Several Plates.

The majority of condensers in practice are composed of more than two plates, as shown in Figure 49, in which case the above equation will read as follows:

$$C=\frac{K(n-1)A}{4.452\times 10^{12}\times d}$$

in which $n$ represents the total number of plates used in the construction of the condenser, and the other letters represent the same quantities as in the first equation.

*Examples.*—1. Determine the capacity of a condenser composed of 200 sheets of tin foil 10 × 10 inches, separated by plate glass .05 inch in thickness, and having a dielectric constant of 5.

*Solution.*—Substituting, in the above equation, the values of the quantities given in the problem gives

$$C = \frac{5 \times (200 - 1) \times 10 \times 10}{4.452 \times 10^{13} \times .05}$$
$$= .000000447 \text{ farads}$$
$$= .447 \text{ microfarads}$$

The above equation may be rewritten with the value of the capacity in microfarads, which gives

$$Q = \frac{C_{m.f.}}{10^6} \times E$$

2. Determine the quantity of electricity stored in a condenser having a capacity of .0001 farad when it is connected to an electrical pressure of 1,000 volts.

*Solution.*—Substituting in one of the above equations gives

$$Q = .0001 \times 1{,}000$$
$$= .1 \text{ coulomb}$$

*Energy Stored in a Charged Condenser.*—Where the diaphragm in Figure 48 is caused to move from its normal position, work is done by the source of the pressure, causing the water to flow through the pipe and, as a result, a certain amount of energy will be stored in the diaphragm. The amount of the energy stored will depend upon the quantity of water passing through the circuit and the average pressure against which this water flows. A better example, perhaps, is that of a standpipe which has been filled with water to a certain height. The work in foot pounds required to fill the tank to the height $H$ feet will be equal to the total weight of water in the tank times the average height, or one-half of $H$.

A similar condition exists in the case of a charged condenser; that is, the energy stored in a charged condenser is equal to the product of the quantity of electricity in the condenser and the average height to which it is raised, or one-half the difference in electrical pressure between its terminals. When the quantity of electricity is expressed in coulombs, the capacity of the condenser in farads, and the difference in electrical pressure between the terminals of the condenser in volts, the energy stored in the condenser, in joules, may be determined by any one of the following equations.

$$W = \tfrac{1}{2}\,QE \text{ joules}$$
$$W = \tfrac{1}{2}\,CE^2 \text{ joules}$$
$$W = \tfrac{1}{2}\,\frac{Q^2}{C} \text{ joules}$$

*Condensers in Series and in Parallel.*—Condensers may be connected in series or in parallel, or in any combination of series and parallel, just as resistances. If two simple plate condensers, $C_1$ and $C_2$, be connected in parallel, as shown in Figure 50, it is equivalent to increasing the area of the plates of one of the condensers and, hence, the resultant capacity of the combination will be greater than that of a single condenser. The equation used in calculating the combined capacity of several condensers in parallel is the same as the one used in calculating the combined resistance of several resistances in series, or

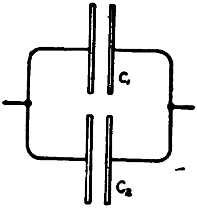


Figure 50.—Condensers in Parallel.

$$C = C_1 + C_2 + C_3 + \dots\dots$$

When condensers are connected in series, as shown in Figure 51, the result is equivalent to increasing the distance between the plates of a single condenser and, hence, the resultant capacity of the combination will be less than that of a single condenser. The equation used in calculating the combined capacity of several condensers in series is the same as the one used in calculating the combined resistance of several resistances in parallel, or

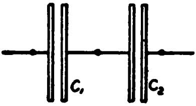


Figure 51.—Condensers in Series.

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots\cdots$$

The difference in electrical pressure between the terminals of a number of condensers all connected in parallel is the same, and the quantity in each is equal to the pressure between its terminals multiplied by the capacity of the condenser. The total quantity stored in the combination is equal to the sum of the quantities in the different condensers.

When condensers are connected in series, the same quantity will be stored in each, and the difference in pressure between the terminals of the different condensers will be to each other inversely as the capacities of the condensers; that is, the greatest difference in pressure will exist between the terminals of the condenser having the smallest capacity, etc.

The total energy stored in any combination of condensers when charged to a definite electrical pressure may be calculated by first determining the capacity of the combination and then proceeding as though it were a single condenser.

*Leakage and Absorption in a Condenser.*—No dielectric is a perfect insulator and, as a result, there will be a current through the dielectric of a condenser when there is a difference in electrical pressure between the terminals of a condenser. As a result of this leakage between the plates, no condenser will retain its original charge when disconnected from the source of electrical pressure, but the quantity stored in the condenser will grow less and less. The resistance of some dielectrics is a great deal higher than that of others and condensers constructed with these dielectrics between the plates will have a small leakage and will retain a given charge for quite a time without any appreciable decrease in its value.

If a charged condenser be discharged by connecting its terminals momentarily with a conductor and this connection is then broken, the condenser will appear to be completely discharged, but after a short time a charge again appears and there is a difference in electrical pressure between the terminals. The pressure in the second case, however, is much less than in the first. The charge which appears after the first discharge is called the *residual charge* and the phenomenon resulting in this charge is called *electric absorption*. The absorption in dielectrics is supposedly due to impurities in the material or a lack of homogeneity.

*Capacity of Coaxial Cylinders.*—The capacity of a condenser formed by placing a conductor inside a tube, as in the case of cables laid in metal sheaths, may be calculated by means of the same equation as that used for the plate condenser. Imagine a cross-section of the conductor and lead sheath as shown in Figure 52. The thickness of the insulation between

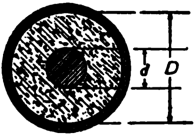


Figure 52.—Cross-section of a Cylindrical Condenser.

the conductor and the sheath will correspond to the distance between the plates; and the average of the area of the outside of the conductor and the area of the inside of the sheath will correspond to the area of the plates.

*Example.*—If a conductor $d_1$ inches in diameter and $l$ inches in length is surrounded by an insulating material having a dielectric constant $K$ and a lead sheath $D$ inches inside diameter, compute the capacity of the combination.

*Solution.*—

$$A = \frac{(D - d_1)}{2} \times l \text{ square inches}$$

$$\text{distance between plates } d = \frac{D - d_1}{2} \text{ inches}$$

$$\text{capacity } C = \frac{K \times A}{4.452 \times 10^{12} \times d}$$

$$= \frac{K \times \dfrac{(D - d_1)}{2} \times l}{4.452 \times 10^{12} \times \dfrac{(D - d_1)}{2}} \text{ farads}$$

A more common equation, however, for the capacity of coaxial conductors and one which is more accurate than the above is as follows:

$$C = \frac{7.354 \times K \times l}{\log_{10} \dfrac{D}{d} \times 10^6} \text{ microfarads}$$

In which $K$ is the dielectric constant of the insulating material; $l$ is the length of the cylinders in feet; $D$ is the inside diameter of the lead sheath, and $d$ is the outside diameter of the conductor. $D$ and $d$ must both be expressed in the same units.

If the insulation between the conductor and the sheath is composed of several different kinds of materials having different dielectric constants, infinitely thin plates may be imagined as existing between the different materials thus forming a number of condensers and they will be connected in series. The capacity of each condenser should be calculated separately by means of the above equation and the results added by means of the equation for condensers in series.

*Capacity of Parallel Round Wires.*—The capacity of a condenser formed by two parallel round wires $d$ units in diameter and $D$ units between centers may be computed by means of the following equation with an error of less than .1 per cent when $D$ is greater than 10$d$.

$$C = \frac{3.677 \times K \times l}{\log_{10} \frac{2D}{d} \times 10^6} \text{ microfarads}$$

in which $l$ is the length of the circuit in feet. In substituting in the equation, $D$ and $d$ must both be in the same units.

*Capacity of a Round Wire Parallel to the Earth.*—The capacity of a round wire $d$ units in diameter and $H$ units above the surface of the earth may be computed by means of the following equation:

$$C = \frac{7.354 \times K \times l}{\log_{10} \frac{4H}{d} \times 10^6}$$

in which $l$ is the length of the wire in feet. In substituting in the equation, $H$ and $d$ must both be expressed in the same units.

*Measuring Capacity by Comparison.*—An unknown capacity may be measured by comparing the quantity discharged from it as compared to that from a standard condenser when they are both charged to the same electrical pressure. The scheme of connections for such a measurement is shown in Figure 53 in which $C$ represents the standard condenser; $X$ the unknown capacity; $G$ the ballistic galvanometer; $P$ a potential rheostat; $S$ a single-pole double-throw switch; and $K$ a short-circuiting key for the galvanometer. Assuming both condensers are completely discharged and that the potential rheostat is adjusted to give a readable deflection on the voltmeter, then there will be a deflection of the moving system of the galvanometer when the switch *S* is closed in the upper position due to the charging quantity in the condenser *C*. Another deflection will be obtained when the switch *S* is closed in its lower position due to the charging quantity in the condenser *X*. If the deflections of the galvanometer are directly proportional to the quantities producing them, then the two quantities in the above cases will bear the same relation to each other as exists between the two deflections.

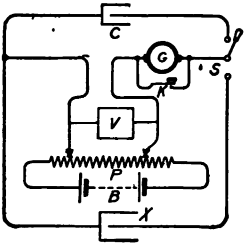


Figure 53.—Comparison Method of Measuring Capacity.

Since the quantity input to a condenser varies directly as its capacity, the applied pressure remaining constant; then, in this case, the capacities of the two condensers are to each other directly as the deflections of the galvanometer.

Very frequently the deflection of the galvanometer does not vary directly as the quantity of electricity sent through it, or, in other words, its constant is not the same for all deflections of the moving system. If this happens to be the case, the comparison can be made as follows: First, charge the unknown capacity to a known pressure and carefully observe the deflection of the galvanometer produced by the charging quantity; and second, charge the known capacity to such a pressure that the charging quantity will produce the same deflection as in the case of the unknown capacity, which will have to be determined by several trials. The condensers should both be completely discharged before attempting to measure the charging quantity. Since the same deflections are produced by the charging quantities, it follows that the quantities themselves are equal, and the capacities of the two condensers will be to each other universally as the electrical pressures required to produce the equal quantities, that is, the larger pressure will be required between the terminals of the smaller capacity. Representing the pressures over the standard, or condenser of known capacity, by $E_c$, and the pressure over the capacity of unknown value by $E_x$, the following relation may be written:

$$\frac{X}{C}=\frac{E_c}{E_x}$$

$$X=\frac{E_c}{E_x}C$$

The value of $X$ will be in farads or microfarads, depending upon what unit the capacity of the standard condenser $C$ is measured in. The choice of the above methods will depend entirely upon the characteristics of the galvanometer and the accuracy with which the pressure impressed upon the terminals of the condensers may be measured.

*Measuring Capacity by the Bridge Method.*—This method of measuring capacity is very similar to the Wheatstone-bridge method of measuring resistances. In this case two arms of the bridge are composed of a known and unknown capacity $C$ and $X$, as shown in Figure 54, while the remaining two arms are composed of known resistances $A$ and $B$, one or both of which may be varied. A source of alternating current $G$ is substituted for the battery, and a telephone receiver

*R* for the galvanometer. The source of alternating current may be nothing more than an ordinary hand magneto. A balance is obtained by adjusting the resistances *A* and *B* until no hum, or a minimum hum, is heard in the telephone receiver with the magneto *G* in operation and the key *K* closed. The relation of the two capacities will be the inverse of the relation between the resistances when a balance is obtained, or

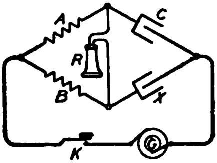


Figure 54.—Bridge Method of Measuring Capacity.

$$\frac{X}{C}=\frac{A}{B}$$

and

$$X=\frac{A}{B}C$$

In measuring capacity by the bridge method, it is necessary that the resistances *A* and *B*, forming two arms of the bridge, be non-inductively wound and that they have no appreciable capacity. The value of the capacity of the condenser *X* will be given in the same unit as the capacity of the standard condenser *C*.

# CHAPTER V

## THE ALTERNATING-CURRENT CIRCUIT

*Definition of an Alternating Electromotive Force or Current.*—An alternating electromotive force is one that changes in value and reverses in direction at certain regular intervals. Such an electromotive force would be induced in a loop of wire revolved in a magnetic field as shown in Figure 55. An alternating current would be produced in any closed circuit connected to brushes which rest upon two metal rings connected to the terminals of the loop.

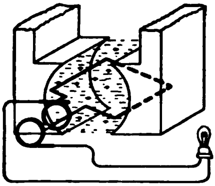


Figure 55.—Single Loop Revolving in a Magnetic Field.

When the plane of the coil is perpendicular to the direction of the field, the two sides are moving parallel to the direction of the field and there will be no induced electromotive force. On the other hand, when the plane of the coil is parallel to the direction of the magnetic field, the two sides are moving perpendicular to the field or directly across it and the induced electromotive force will have its maximum value. The induced electromotive force for intermediate positions of the coil will be equal to the maximum induced electromotive force multiplied by the sine of the angle the plane of the coil makes with a plane perpendicular to the direction of the magnetic field.

*Hydraulic Analogy of an Alternating Current.*—The alternating current in an electrical circuit can be compared to the flow of water in a closed pipe connected to a cylinder, in which there is an oscillating piston operating, as shown in Figure 56. If the arm *A* be revolved at a constant angular velocity, there will be a flow of water through the pipe, which may be represented graphically by a curve, as shown in Figure 57. The length of the horizontal line corresponds to the time of one complete revolution of the arm *A;* and the length of the vertical line from the horizontal one to the curve—called the ordinate of the curve—at any instant, represents the rate at which the water is passing a given cross-section of the pipe, or the current in the pipe. As one direction of flow is assumed positive, the other is negative, and this is represented in the curve by drawing part of the curve above the horizontal line and part below. The positive and negative directions are only relative. From

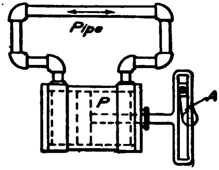


Figure 56.—Hydraulic Analogy of an Alternating-Current Circuit.

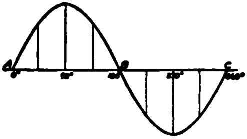


Figure 57.—Curve Representing Flow of Water.

an inspection of Figure 56 it will be obvious that the movement of the piston in the cylinder is proportional to the horizontal component of the motion of the pin in the end of the arm, and this component of the pin's motion is proportional to the sine of the angle between the arm and the horizontal. Such a curve as that shown in Figure 57 is called a *sine curve.*

*Cycle, Frequency, Alternation, Period, Synchronism, and Phase Displacement.*—When an alternating electromotive force, or current, has passed through a complete set of positive and negative values, starting from any value and again returning to that value in the same direction, it has completed what is called a *cycle.* A complete set of positive and negative values, or one cycle, is shown by the curve in Figure 57.

The number of cycles the electromotive force, or current, passes through in one second is called the *frequency.* Thus, a 25-cycle electromotive force or current would be one that passed through a complete set of positive and negative values 25 times per second.

An *alternation* is half a cycle, and corresponds to a complete set of positive or negative values of electromotive force, or current. There will be just twice as many alternations in a given time as there are cycles, or a frequency of 25 cycles would mean 50 alternations per second.

The *period* of an electromotive force, or current, is the time in seconds required to complete one cycle. Thus, the period of a 25-cycle electromotive force, or current, would be 1/25 of a second and of a 60-cycle electromotive force, or current, 1/60 second.

Two electromotive forces, or currents, are said to be in *synchronism* when they have the same frequency.

Any number of electromotive forces, or currents, are said to be in *phase* when they pass through corresponding values of their respective cycles at the same time. The two curves shown in Figure 58 do not pass through zero or their maximum points at the same time and, therefore, are said to be displaced in phase. Assume that curve *E*, Figure 58, represents the electromotive force acting on a circuit, and that curve *I* represents the current produced by this electromotive force. The current in this case passes through zero value after the electromotive force, or it is said to lag the electromotive force in phase. This phase displacement of two quantities with respect to each other may be measured in degrees or time. The total length of the line *AC*, Figure 58, corresponds to one cycle of the electromotive force, or 360 degrees, and likewise the length of the line *DG*, which is equal to *AC*, corresponds to one cycle of the current, each starting from zero value. The two curves are displaced in phase the same fractional part of 360 degrees as the length of line *AD* is a part of *AC*. The displacement of the two curves measured in terms of time is equal to such a fractional part of the period as the length of line *AD* is a part of *AC*.

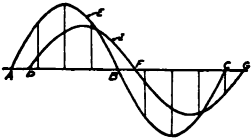


Figure 58.—Two Curves Displaced in Phase.

*Maximum, Average, and Effective Values of Electromotive Force, or Current.*—The maximum value of an alternating electromotive force, or current, is the value represented by the ordinate of the electromotive force, or current, curve having the greatest length. Thus in Figure 57 the maximum flow occurs at 90° and 270°, it being opposite in direction for the two positions but having the same value.

The average value of an alternating electromotive force, or current, is equal to the average of all of the instantaneous electromotive forces, or currents, for a complete alternation, starting with zero value and returning to zero. For a sine curve the average values of the electromotive force and current is .636 times their maximum value. This relation is determined by finding the area of a positive or negative loop of the electromotive force, or current, curve and dividing this area by the distance between the two points where the curve crosses the horizontal line. Each of the rectangles shown by shaded portions in Figure 59 have the same area as one loop of the sine curve, and the altitude of these rectangles is equal to .636 of the maximum ordinate of the curve.

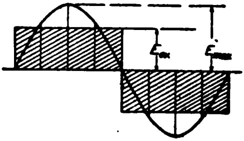


Figure 59.—Relation of Maximum and Average Values.

The effective value of an alternating current is numerically equal to a steady direct current that will produce the same heating effect in a given time as is produced by the changing alternating current in a like time. If a conductor has a resistance of *R* ohms and it is carrying a current of *I* amperes, then the power in watts expended in heating the conductor is equal to the value of the current squared, times the resistance of the conductor. When the conductor is carrying an alternating current, the heating effect is continuously changing and it is necessary to add the instantaneous heating effects for a given time in order to get the total heating effect. The same heating effect could be produced by a steady current and the value of the steady current that must flow for the same time as the alternating current in order to produce the same total heating effect corresponds in value to the effective alternating current

$$\begin{aligned} I^2 Rt &= \text{average } i^2 Rt \\ I^2 &= \text{average } i^2 \\ I &= \sqrt{\text{average } i^2} \end{aligned}$$

The small letters in the above equations are used to represent instantaneous values.

For a sine curve, the square root of the average of the instantaneous currents squared is equal to .707 times the maximum current. The ordinate of the rectangles shown in Figure 60 corresponds to the effective value of a current whose maximum value corresponds to the maximum ordinate of the sine curve.

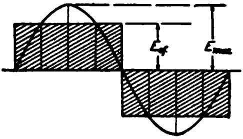


Figure 60.—Relation of Maximum and Effective Values.

The effective value of an electromotive force has the same relation to its maximum value as exists between maximum and effective currents, namely, the effective electromotive force for a sine curve is equal to .707 times the maximum value.

Alternating-current ammeters and voltmeters indicate the effective value of current and electrical pressure.

$$\begin{aligned} \text{Average value} &= .636 \text{ maximum value} \\ \text{Effective value} &= .707 \text{ maximum value} \\ \text{Effective value} &= 1.11 \text{ average value} \end{aligned}$$

The form factor of a wave is numerically equal to the effective value divided by the average value. For a sine wave it is 1.11. A wave having more of a peak than a sine wave will have a form factor greater than

1.11, and a wave which is flatter on top than the sine wave will have a form factor less than 1.11. The minimum value any form factor can have is unity, and in such a case the loops of the wave are rectangular in form. In the following calculations the current and the electromotive force are assumed to follow a sine curve.

*Vector Representation of Alternating Electromotive Forces and Currents.*—Since electromotive forces and currents have both direction and magnitude, their relation to each other may be represented by a vector diagram. Thus, if a current of 10 amperes lagging the electromotive force producing it, which has a value of 10 volts, by an angle of 45 degrees would be represented as shown in Figure 61. The vectors can be thought of as rotating, and one revolution corresponding to 360 degrees. In such vector diagrams the counter-clockwise direction of rotation is taken as positive. Some one of the vectors in the diagram is taken as a reference line and the others are so drawn that they make the proper angle with this line and also with each other. In all practical cases, the effective values of the electromotive force and the current are used in vector diagrams.

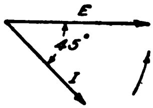


Figure 61.—Vector Diagram.

*Ohm's Law for an Alternating-Current Circuit.*—Ohm's law, as applied to the direct-current circuit, expresses the relation between the electrical pressure between any two points of a circuit, the resistance of the circuit between the two points, and the current in the circuit. In such a case the only opposition offered by the circuit to the flow of electricity is the resistance of the materials composing the circuit. When an alternating electrical pressure acts upon a circuit, the effective current produced is equal to the effective pressure divided by the resistance of the circuit, provided there is no capacity or inductance in the circuit. If capacity or inductance, or both, be present in the circuit, the total opposition offered by the circuit to the flow of electricity through it will be greater than the ohmic resistance of the circuit unless the effects of capacity and inductance neutralize each other, as will be explained later, in which case the opposition offered is equal to the ohmic resistance of the circuit. The combined effects of resistance, capacity, and inductance in opposing the free flow of electricity through a circuit is called the *impedance* of the circuit. The combined effects of capacity and inductance is called the *reactance* of the circuit; the effect of capacity is called *capacity reactance* and the effect of inductance is called *inductive reactance.* Reactance and impedance are measured in the same unit as resistance, namely, the ohm. The symbol usually used for impedance is the letter $Z$, and that for reactance the letter $X$, it being given a subscript $C$ when it represents capacity reactance, and a subscript $L$ when it represents inductive reactance.

Ohm's laws for the alternating-current circuit may be written as follows:

$$\text{effective current} = \frac{\text{effective pressure}}{\text{impedance}}$$

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}}$$

$$I = \frac{E}{Z}$$

The methods of calculating the capacity and inductive reactance and the method of combining the reactances and resistance to obtain the impedance will be given in the following sections.

*Effect of Resistance in an Alternating-Current Circuit.*—In discussing the effect of resistance in this section and the effects of inductance and capacity as given in the following two sections, we will assume that a certain current is to be produced in the circuit and then proceed to calculate the total electrical pressure required to overcome the resistance and the inductive and capacity reactance.

If the current through a circuit containing resistance alone be represented by the curve *I*, Figure 62, then the electrical pressure at any instant required to produce this current is equal to the product of the current at that instant and the resistance of the circuit. It is obvious that the pressure required to produce the desired current varies directly as the current, and they pass through corresponding values at the same time. The current and the electrical pressure will, as a result, be in phase, and the electrical pressure may be represented by a second curve, $E_r$.

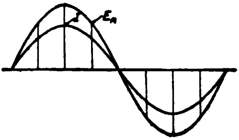


Figure 62.—Phase Relation of E. M. F. and Current in a Circuit Containing Only Resistance.

*Effect of Inductance in an Alternating-Current Circuit.*—Let us assume that the circuit contains inductance alone, and that there is a current to be produced in the circuit, which may be represented by the curve *I*, Figure 63. There will be a magnetic field produced by the current which will vary in value with the current and also reverse in direction when the current reverses. With a change in magnetic flux associated with the circuit, there will be an induced electromotive force produced in the circuit, which at any instance is proportional to the rate at which the magnetic flux is being cut by the circuit. Since the magnetic flux associated with the circuit varies in value with the current, it is apparent that this flux will be changing in value at the greatest rate when the current is changing at the greatest rate or passing through zero value and, as a result, the induced electromotive force in the circuit will be a maximum when the current is a minimum, or zero. It must be clearly understood that the induced electromotive

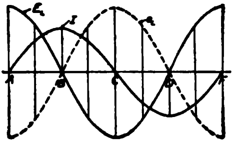


Figure 63.—Phase Relation of E. M. F. and Current in a Circuit Containing Only Inductance.

force does not depend upon the value of the total magnetic flux, but upon the rate at which this flux is changing in value, that is, the rate at which the circuit is cutting the flux. The direction of this induced electromotive force will always be such as to oppose any change in the value of the current. Thus, if the current be increasing in value in the positive direction, as it is between the points *A* and *B* in the figure, the induced electromotive force acts in such a direction as to oppose this change in the current, or it acts in the negative direction. If the current be decreasing in value in the positive direction, as it is between the points *B* and *C* in the figure, the induced electromotive force opposes this decrease, or it acts in the positive direction. If the current be increasing in value in the negative direction, the induced electromotive force will oppose this increase, or it acts in the positive direction, etc. The induced electromotive force produced in the circuit may be represented by a curve, $e_L$, as shown in the figure. Since the circuit was assumed to have only inductance—a theoretical condition, as all circuits will have some resistance—the electromotive force required to produce the desired current would be one that would just overcome the electromotive force induced in the circuit due to the inductance. Such an electromotive force would be represented by the curve $E_L$, whose ordinates are at each instant equal to those of the curve $e_L$ but opposite in sign, or $E_L$ acts opposite to $e_L$ and will produce the current $I$.

From the relation of the curves $E_L$ and $I$ in the figure, it is seen that the current and the impressed electrical pressure in a circuit containing only inductance are displaced in phase by 90 degrees and that the current lags the impressed electrical pressure.

*Effect of Capacity in an Alternating-Current Circuit.*—The current to be produced in a circuit containing only capacity may be represented by a curve such as $I$, Figure 64. The action of the condenser in the alternating-current circuit may be explained by means of the hydraulic analogy shown in Figure 48. Assuming the liquid in the pipe is flowing toward the right, which we will call the positive direction, and that the diaphragm is being deflected to the right of its normal position. When the diaphragm is in its

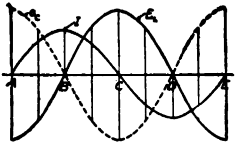


**Figure 64.**—Phase Relation of E. M. F. and Current in a Circuit Containing Only Capacity.

normal, or vertical, position, it offers no opposition to the flow of the liquid; but just as soon as it is moved from this position, it immediately opposes the flow and its opposition increases in value as its deflection increases. The strain in the diaphragm when it is deflected to the right of its normal position acts toward the left; and when it is deflected to the left of its normal position, the strain acts toward the right. The current in such a circuit is a maximum when the diaphragm is in its vertical position, or when the opposition is a minimum; and it is a minimum when the diaphragm is deflected to its maximum deflection, or when the opposition is a maximum. Assuming the current is decreasing from a positive maximum value or the diaphragm is moving from its normal position toward the right, then the strain in the diaphragm is increasing and it acts in the opposite direction to the current. Next, assume the current is increasing from zero value toward a maximum negative value, or the diaphragm is moving from the extreme right toward its normal position, then the strain in the diaphragm is decreasing and it acts in the same direction as the current. Next, assume the current is decreasing in the negative direction, or the diaphragm is moving from its normal position toward the left, then the strain in the diaphragm is increasing and it acts in the opposite direction to the current. Finally, assume the current is increasing from a zero value toward a maximum positive value, or the diaphragm is moving from the extreme left toward its normal position, then the strain in the diaphragm is decreasing and it acts in the same direction as the current. The value of the strain in the diaphragm and the current in the hydraulic circuit correspond in their phase relation to the strain in the condenser dielectric and the current through it, and their phase relation. The strain between the terminals of the condenser, through which a current, *I*, is to be produced, may be represented by a curve, $e_C$. The impressed electrical pressure must at each instant be equal and opposite to the strain in the dielectric in order to produce the current $I$, or it would be represented by a curve $E_C$ whose ordinates are equal to those of $e_C$ but of opposite sign. It is seen from an inspection of the curves that the current in a circuit containing capacity alone leads the impressed pressure by 90 degrees.

*Numerical Value of Electromotive Force to Overcome the Effect of Resistance, Inductance, and Capacity.*—If the current in a circuit is changing according to a sine curve and $I$ be taken as the effective value of current in the circuit, the effective electrical pressure required to overcome the effects of inductance and capacity, represented by $E_L$ and $E_C$, can be determined by the following equations:

$$E_L = 2 \times \pi \times f \times L \times I$$

$$E_C = \frac{I}{2 \times \pi \times f \times C}$$

in which $\pi = 3.1416$; $f$ represents the frequency in cycles per seconds; $L$ is the inductance of the circuit in henries; $C$ is the capacity of the circuit in farads; and $I$ is the effective current in amperes. The electrical pressure required to overcome the effect of inductance leads the current by 90 degrees, and the electrical pressure required to overcome the effect of capacity lags the current by 90 degrees.

The electrical pressure required to overcome the resistance is in phase with the current, and numerically equal to the product of the resistance and effective current in amperes. These three electrical pressures can be represented by three vectors, as shown in Figure 65, the counter-clockwise direction of rotation being taken as positive.

*Total Electrical Pressure Required to Produce a Given Alternating Current.*—If the two quantities

$2\pi fLI$ and $I \div 2\pi fC$, Figure 65, are equal in value, the vectors representing them will be equal in length. The resultant of these two vectors will then be zero, and the only electrical pressure required to produce the current $I$ is that to overcome the resistance, which is equal to $RI$. If, however, the electrical pressure required to overcome the effect of inductance is greater than that required to overcome the effect of capacity, the vector $E_L$, Figure 65, will be greater in length than the vector $E_C$. The resultant of these two vectors will be equal to $E_L - E_C$, and its direction will correspond to the larger vector, or in this case $E_L$. This resultant must now be combined with $E_R$ in order to obtain the value of the total pressure required to produce the current $I$, which can be done graphically, as shown in the figure. The resultant $E$ is equal to the diagonal of a parallelogram whose sides in this case are $(E_L - E_C)$ and $E_R$, and its value is equal to the square root of the sum of the squares of the two sides, or

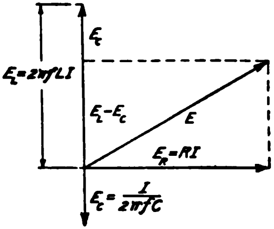


Figure 65.—Vector Diagram of Pressures.

$$E = \sqrt{E_r^2 + (E_L - E_C)^2}$$

Substituting in the above equation the values of $E_L$ and $E_C$ as given in the previous section, and the value of $E_R$, which is equal to $RI$, gives the equation.

$$E = \sqrt{(RI)^2 + \left(2\pi fLI - \frac{I}{2\pi fC}\right)^2}$$

By taking $I$ from under the radical sign, this equation can be changed to the form

$$E = I\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

or

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

The above equation gives the value of the effective current $I$ in amperes in terms of the impressed effective electrical pressure $E$, in volts; the inductance $L$ of the circuit, in henries; the capacity $C$ of the circuit, in farads; the frequency $f$, in cycles per second; and the constant $2\pi$, which equals $2 \times 3.1416$, or 6.2832.

*Numerical Value of Capacity and Inductive Reactance, and Impedance.*—The denominator of the right-hand portion of the last equation in the previous section is the impedance of an alternating-current circuit in ohms. The part under the radical sign which is enclosed in parenthesis, is called the reactance of the circuit, and it is represented by the letter $X$, or

$$X = 2\pi fL - \frac{1}{2\pi fC} \text{ ohms}$$

The above value of $X$ is composed of two parts, $2\pi fL$ and $(1 \div 2\pi fC)$. The product of $2\pi fL$ is the numerical value of the inductance reactance and it is represented by the symbol $X_L$, while 1 divided by $2\pi fC$ is the numerical value of the capacity reactance and is represented by the symbol $X_C$. In general

$$Z = \sqrt{R^2 + X^2}$$

or

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

From the above expression for the numerical values of the reactances, it is really seen that the value of the inductive reactance increases with an increase in frequence and the value of the capacity reactance decreases with an increase in frequency.

*Phase Relation of the Electrical Pressure over the Various Parts of a Series Circuit and the current in the Circuit.*—The series circuit shown in Figure 66 is composed of three parts, a resistance *R*, an inductance *L*, and a capacity *C*. Four alternating-current voltmeters are indicated in the figure, one being connected across each of the three elements and the fourth across the entire combination. The current is the same in all parts of such a circuit, neglecting the current through the voltmeters, and an ammeter, that will operate on alternating current, may be connected in the circuit at any point and it will indicate the current that exists in the circuit. The three voltmeters marked $V_1$, $V_2$, and $V_3$, indicate the electrical pressure between the terminals of the resistance, inductance, and capacity, and the one marked *V* indicates the total pressure over the entire circuit. In the case of a series circuit carrying a direct current, the sum of the pressures over the various parts is equal to the total impressed pressure; but this is not the case for a series circuit carrying an alternating current unless the elements of the circuit are of the same kind, namely, resistances, inductances, or capacities, or similar combinations of two or more of these. The drop in pressure between the terminals of the resistance is in phase with the current; the drop in pres-

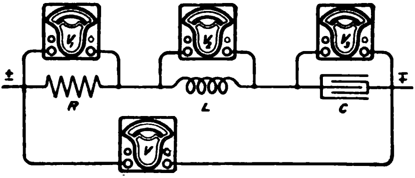


Figure 66.—Series Circuit.

sure between the terminals of the inductance leads the current by 90 degrees; and the drop in pressure between the terminals of the capacity lags the current by 90 degrees. The difference in electrical pressure between the terminals of the various parts of a series circuit must be added vectorily in order to obtain the total electrical pressure over the entire combination.

In practice, it is impossible to have a series circuit composed of elements which in turn are composed of inductance and capacity alone, and the various elements are usually a combination of resistance and inductance, or capacity, or both. In such a case the drop in pressure between the terminals of the different elements is never displaced in phase from the current by 90 degrees, but may approach this displacement very closely in some cases. It is possible to have an element composed of resistance alone and in such case the current through the resistance and the pressure between its terminals are in phase.

*Phase Relations of the Currents in the Different Branches of a Divided Circuit and the Electrical Pressure Acting on the Circuit.*—A divided or parallel circuit composed of three branches is shown in Figure 67. The upper branch is a non-inductive resistance,

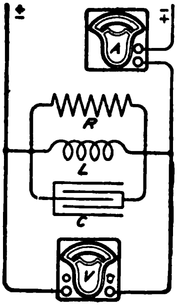


Figure 67.—Divided Circuit.

the middle branch is an inductance, and the lower branch is a capacity. A voltmeter *C* connected between the terminals *B* and *D* indicates the difference in pressure between the terminals of the three branches. The current in any one of the branches is equal to the pressure acting on the branch, as indicated by the voltmeter, divided by the impedance of the branch. In the upper branch, the pressure and the current are in phase, since there is no reactance present in the circuit. The current lags the pressure by 90 degrees in the middle branch and leads the pressure by 90 degrees in the lower branch, on account of the presence of only inductive and capacity reactance in the middle and lower branches, respectively. If the currents in the three branches were all in phase with the electrical pressure, or if they were each displaced in phase with respect to the electrical pressure the same amount, the current in the main circuit would be equal to the numerical sum of the different branch currents. The above phase relation between branch currents and the pressure does not, as a rule, exist in practice and it is necessary to add the different branch currents by means of vectors. It is impossible in practice to have a branch of a divided circuit composed of inductance, or capacity, alone, and the various branches are usually combinations of resistance and inductance, or capacity, or both. In such a case the current in any branch and the pressure acting on the branch are never displaced in phase by 90 degrees, but may approach this displacement very closely in some cases. It is possible to have one branch of a divided circuit composed of resistance alone, and in such a case the current and the pressure are in phase.

*Impedance Diagram.*—Since the impedance of a circuit is equal to the square root of the sum of the squares of the resistance and the reactance of a circuit, the relation of the three quantities may be expressed graphically by means of a right-angled triangle, as shown in Figure 68. The vertical side represents the reactance, the horizontal side the resistance, and the diagonal represents the impedance.

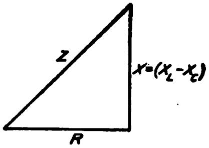


Figure 68.—Impedance Diagram.

*Combining Impedances in Series.*—The combined impedance of several impedances in series may be obtained by combining their impedance diagrams, as shown in Figures 69 and 70, respectively. In the first case both reactances are inductive and the resultant reactance is equal to the sum of the reactances of the several impedances. Likewise, if both reactances had been due to capacity, the resultant reactance would be equal to the sum of the several reactances, but the lines representing the reactances would then be drawn downward from the horizontal rather than upward. In the second case the two reactances are different, one being due to inductance and the other to capacity, and the resultant reactance is equal to the difference

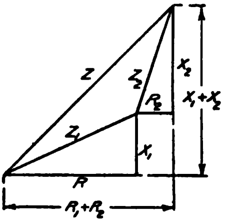


Figure 69.—Adding Impedances with Like Reactances.

in their values. It is customary in dealing with such problems to consider the capacity reactance as negative and the inductive reactance as positive, the resultant reactance being the algebraic sum of the reactances. Then if the resultant reactance is positive, the current lags the electrical pressure; and if the resultant reactance is negative, the current leads the electrical pressure.

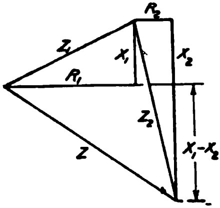


Figure 70.—Adding Impedances with Unlike Reactances.

The impedance $Z$ of a series circuit composed of a number of impedances $Z_1$, $Z_2$, $Z_3$, etc., can be calculated by substituting in the following general equation

$$Z=\sqrt{(R_1+R_2+R_3+\text{etc.})^2+(X_1+X_2+X_3\ \text{etc.})^2}$$

in which $R_1$, $R_2$, $R_3$, etc., represent the resistances of the different impedances, and $X_1$, $X_2$, $X_3$, etc., represent their reactances.

*Example.*—Calculate the impedance of a circuit composed of two impedances in series having resistances of 10 and 15 ohms and reactances of $-12$ and 20 ohms, respectively.

*Solution.*—The total resistance of the circuit is equal to $(10+15)$, or 25 ohms, and the resultant reactance is equal to $(-12+20)$, or 8 ohms. Combining the total resistance and resultant reactance gives

$$\begin{aligned} Z &= \sqrt{25^2+8^2} \\ &= \sqrt{689} \\ &= 26.2\text{-}\ \text{ohms} \end{aligned}$$

The current in this circuit due to any electrical pressure would be equal to the value of the effective pressure, in volts, divided by the impedance and the current would lag the pressure because the resultant reactance is positive.

The impedance of a series circuit can be determined experimentally by sending an alternating current of known value through the circuit and measuring the difference in pressure between the terminals of the circuit, which, divided by the current, gives the impedance. The frequency of the current should correspond to the frequency at which the impedance is desired.

*Combining Impedances in Parallel.*—In calculating the combined impedance of a number of impedances in parallel, an equation is used similar to that for impedances in series, but instead of using the separate resistances and reactances, the *conductance* and *susceptance* of the circuit are used.

The conductance of a circuit is a quantity by which the electrical pressure must be multiplied to give the component of the current parallel to the electrical pressure. The conductance is represented by the letter $G$ and it is numerically equal to

$$G=\frac{R}{R^2+X^2}=\frac{R}{Z^2}$$

The susceptance of a circuit is a quantity by which the electrical pressure must be multiplied to give the component of the current perpendicular to the electrical pressure. The susceptance is represented by the letter $B$ and it is numerically equal to

$$B=\frac{X}{R^2+X^2}=\frac{X}{Z^2}$$

The reciprocal of the impedance is called the *admittance* and it is represented by the letter $Y$.

$$Y = 1 \div Z$$

Since $Z = E \div I$, then

$$Y = I \div E$$

Conductance, susceptance, and admittance are all measured in a unit called the *mho.*

The admittance of a circuit bears the same relation to the conductance and susceptance as exists between the impedance; resistance, and reactance, or

$$Y = \sqrt{G^2 + B^2}$$

The total impedance of a number of impedances in parallel, then, is equal to the reciprocal of the admittance, which can be determined by the equation

$$Y = \sqrt{(G_1 + G_2 + \text{etc.})^2 + (B_1 + B_2 + \text{etc.})^2}$$

*Example.*—Calculate the total impedance of two impedances in parallel, having resistances of 6 and 8 ohms and reactances of 4 and 5 ohms, respectively.

*Solution.*—Substituting in the above equations for conductance and susceptance gives

$$G_1 = \frac{6}{6^2 + 4^2} = \frac{6}{52} = .1154 \text{ mho}$$

$$G_2 = \frac{8}{8^2 + 5^2} = \frac{8}{89} = .0899 \text{ mho}$$

$$B_1 = \frac{4}{6^2 + 4^2} = \frac{4}{52} = .0769 \text{ mho}$$

$$B_2 = \frac{5}{8^2 + 5^2} = \frac{5}{89} = .0562 \text{ mho}$$

The total conductance $G$ of the circuit is equal to $G_1 + G_2$, or

$$G = .1154 + .0899 = .2053 \text{ mho}$$

and the total susceptance $B$ of the circuit is equal to $B_1 + B_2$, or

$$B = .0769 + .0562 = .1331 \text{ mho}$$

The admittance may be calculated by substituting in the equation which gives its value in terms of the total conductance and total susceptance, or

$$Y = \sqrt{.2053^2 + .1331^2}$$
$$= .244 \text{ mho}$$

The impedance is equal to the reciprocal of the admittance, or

$$Z = 1 \div .244$$
$$= 4.09 \text{ ohms}$$

The impedance of several impedances in parallel may be determined by sending a current through the combination and noting the drop in electrical pressure between the terminals, which, divided by the current, gives the impedance. It must always be kept in mind that the impedance depends upon the frequency and, in making a measurement as indicated above, the proper frequency should be used.

*Instantaneous Power in an Alternating-Current Circuit.*—The instantaneous power in an alternating-current circuit at any instant is equal to the product of the current in the circuit at that instant and the electrical pressure acting on the circuit at that instant. The two curves $I$ and $E$, Figure 71, represent the current in a circuit and the pressure acting on the circuit, both being drawn to a suitable scale. These two curves are in phase and the ordinates of both will always be either positive or negative, which results in their product always being positive. A third curve may be drawn having ordinates proportional to the product

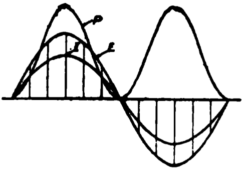


Figure 71.—Instantaneous Power, Current and Pressure in Phase.

of the quantities represented by the ordinates of the curves *I* and *E*, as indicated by *P* in the figure.

If the current and the electrical pressure be displaced in phase, as shown in Figure 72, the ordinates of the two curves do not change in sign at the same time and, as a result, the product of the quantities represented by these ordinates is not positive throughout the cycle but is negative in sign for a portion of the time, which results in a part of the curve *P* being below the horizontal line. Therefore, when the current and the pressure are in phase, or there is no resultant reactance in the circuit, the power is all positive, that is, no power is being returned from the generator. The power in such a case is proportional to the combined area of all the loops in the power curve.

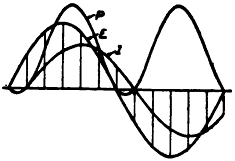


Figure 72.—Instantaneous Power, Current and Pressure Out of Phase.

If the current and the pressure are not in phase, or the resultant reactance in the circuit is not zero, the effective power is proportional to the difference in area of the positive and the negative loops of the power curve, because the negative loops of the power curve represent power which is being returned to the generator.

*True Power in Alternating-Current Circuit.*—The true power in an alternating-current circuit in watts, in which the current and the electrical pressure are in phase, is equal to the product of the effective values of the pressure and the current. The true power in an alternating-current circuit in which the current and the electrical pressure are displaced in phase by 90 degrees is zero, because the areas of the positive and the negative loops of the power curve are equal and just as much power is returned to the generator from the circuit as the generator delivers to the circuit. It is impossible to have such a condition in practice as all circuits contain some resistance.

In general, the current and the electrical pressure in an alternating-current circuit are displaced in phase by angles varying in value between zero and 90 degrees, depending upon the relative amounts of reactance and resistance present in the circuit. In any case the actual current in the circuit may be thought of as composed of two parts or components, one of which is in phase with the pressure and the other at right angles to the pressure. It is obvious from the previous discussion that there is no resultant power due to the component of the current at right angles to the pressure, as the positive and the negative loops of the power curve are equal in area. The power due to the component of the current in phase with the pressure is all positive or, if this were the only current in the circuit, the generator would be delivering power all the time. The component of the current at right angles to the pressure is called the *reactive component,* while the component in phase with the pressure is called the *power component.* The effective power due to the resultant, or actual, current in the circuit is equal in amount to that due to the power component of the current. Hence, in order to compute the effective power in such a circuit, it is necessary to determine the value of the component of the current in the circuit in phase with the pressure, and then multiply the effective value of this component by the effective pressure.

A current and an electrical pressure are represented by two vectors $I$ and $E$, Figure 73, and they are displaced in phase by 30 degrees, the current lagging the pressure. The component of the current in phase with the electrical pressure, or the power component, is proportional to the projection of the current vector upon the pressure vector. The component of the current at right angles to the pressure, or the reactive component, is proportional to the projection of the current vector upon a line perpendicular to the pressure vector. These projections are shown graphically in the figure, but their numerical values may be determined as follows: The power component of the current is equal to the product of the current and the cosine of the angle $\theta$ between the current and the pressure, while the reactive component of the current is equal to the product of the current and the sine of the angle $\theta$ between the current and the pressure. The effective power, then, is equal to

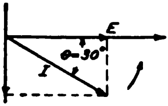


**Figure 73.—Vector Diagram.**

$$P = E \times I \cos \theta$$

The product of $E$ and $I$ in the above equation is called the *apparent power,* and the term $\cos \theta$ is called the *power factor,* since it is the factor by which the apparent power must be multiplied in order to obtain the true or effective power. When the current and the pressure are in phase, the angle between them is zero and the cosine of $\theta$ is equal to unity, which results in

$$P = E \times I \times 1 = E \times I$$

If the current and pressure are displaced in phase by 90 degrees, the angle between them is zero and the cosine of $\theta$ is equal to zero, which results in

$$P = E \times I \times 0 = 0$$

The power factor of a circuit will depend upon the relative amounts of resistance and reactance present in the circuit. Thus, induction motors running on light load take a lagging current, while synchronous motors with over-excited field take a leading current.

In transmitting energy over a circuit, it is always desirable to have the value of the power factor as near unity as possible, since the current in the circuit for a given power is a minimum with unity power factor and, hence, the losses are a minimum. Frequently special equipment is introduced or connected to the circuit in order to increase the value of the power factor, the saving resulting in the decrease in the losses offsetting the cost of the equipment.

*Single-, Two-, and Three-Phase Alternators.*—A single loop of wire revolving in a magnetic field, as shown in Figure 55, corresponds to what is called a *single-phase alternating-current generator.* There is only one electromotive force induced and the terminals of the loop may be connected to the external circuit by brushes and slip rings as shown in the figure.

If a second loop of wire be mounted on the shaft with the first with its plane at right angles to the first, the electromotive forces in the two loops will be displaced in phase by 90 degrees. Each loop may be provided with two slip rings and connected to independent external circuits or one ring may be common to both loops in which case only three rings and three brushes are required. These two possible arrangements are shown diagrammatically in Figures 74 and 75. This combination of two loops displaced in phase by 90 degrees constitutes what is termed a *two-phase alternator.*

If three loops of wire, whose planes are 60 degrees apart, be revolved in a magnetic field, there will be electromotive forces induced in the loops which will be displaced in phase from each other by 120 degrees.

Each of these three loops may be provided with two slip rings and two brushes and connected to independent external circuits, as shown in Figure 76, or each ring may be common to two loops, in which case only three rings and three brushes will be needed. The three loops may be connected to the three rings in two different ways, as shown in Figures 77 and 78. The loops are said to be delta-connected in Figure 77, and star-connected in Figure 78. The delta connection is usually represented by the symbol △, and the star connection by the letter Y.

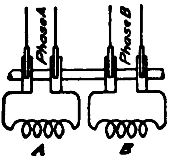


Figure 74.—Four-Ring Two-Phase Alternator.

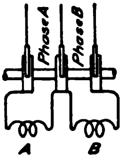


Figure 75.—Three-Ring Two-Phase Alternator.

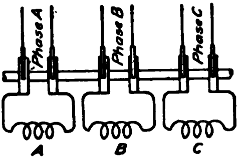


Figure 76. — Six-Ring Three-Phase Alternator.

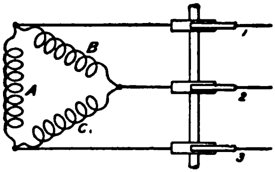


Figure 77.—Delta-Connected Three-Phase Alternator.

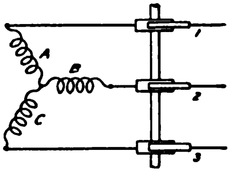


Figure 78.—Star-Connected Three-Phase Alternator.

*Producing a Rotating Magnetic Field by Currents Displaced in Phase by 90°, or Approximately That Amount.*—An iron ring with two continuous windings about it is shown in Figure 79. Each of these windings is supplied with current from a two-phase alternator, the connections being made as indicated in the figure. Let us assume that the current in the winding connected to phase *A* is at its maximum value and then the current in the winding connected to phase *B* will be zero. Assume the current in the winding connected to phase *A* is passing through the winding from terminal *A* to terminal *B*, then there will be a south magnetic pole formed at *A* and a north magnetic pole at *B*. Next assume that the current in the winding connected to phase *B* flows through the winding from the terminal *C* to the terminal *D* as soon as the current in phase *B* passes through zero value. The current in phase *B* tends to produce a south magnetic pole at the terminal *C* and a north magnetic pole at the terminal *D*. Now as the current in phase *B* increases in value, its magnetic effect increases and at the same time the current in phase *A* decreases in value, its magnetic effect decreases, and this action continues for one-fourth of a cycle, during which time the south magnetic pole is moving from *A* to *C* and the north magnetic pole is moving from *B* to *D*. It must be clearly understood that each current does not produce independent magnetic poles only when the value of the other current is zero, but the magnetic effects of the two currents in the separate windings combine and produce a resultant magnetic effect whose direction determines the location of the poles on the ring and their polarity, and whose value determines their strength.

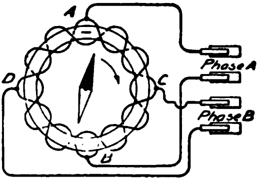


Figure 79.—Producing a Rotating Magnetic Field.

At the end of the first quarter of the cycle, the current in phase *A* is zero and it then reverses in direction, while the current in phase *B* is a maximum and it starts to decrease, which results in the south pole moving from *C* toward *B* and the north pole moving from *D* toward *A*. At the end of the half cycle the current in phase *B* reverses and that in phase *A* decreases, while at the end of three-fourths of the cycle, the current in *A* reverses and the current in *C* decreases to zero. If a compass needle were placed inside such a ring, it would revolve in a clockwise direction due to the action of the magnetic poles produced by the currents. The direction of rotation of the compass needle can be reversed by reversing the connection of one of the windings which will reverse the direction of its magnetic effect and thus cause the poles, produced by the currents, to travel around the ring in the reverse direction to what they did in the first case.

A rotating magnetic field may be produced by a single source of alternating electromotive force in a manner similar to the above by connecting the two windings in parallel and having the inductance of one winding very high and the inductance of the other as low as possible. The difference in the inductances will cause the currents in the two windings to be displaced in phase by approximately 90 degrees and a rotating magnetic field will be produced. The direction of rotation of this field can be changed by reversing the connections of one of the windings. This method of producing a rotating magnetic field is used in the construction of small alternating-current motors and watt-hour meters. The construction in the case of the motor or meter is not the same as in the ring but the fundamental principle is the same.

If a disk of metal be mounted above or below the ring shown in Figure 79, there will be induced currents produced in the disk due to the movement of the field with respect to the disk and, as a result of the action of these currents, there will be a torque produced which will cause the disk to rotate in the direction the magnetic field is rotating.

# CHAPTER VI

## CALCULATION AND MEASUREMENT OF RESISTANCE

### CALCULATION OF RESISTANCE

*Resistance.*—Resistance is defined as being a property of materials which opposes the free flow of electricity through them, but does in no way tend to cause a current in the opposite direction to that in which the electricity actually moves. The unit in which resistance is measured is the ohm, and it is numerically equal to the resistance offered by a uniform column of mercury 106.3 centimeters in length and 14.4521 grams in mass, at the temperature of melting ice. The cross-section of such a column is one square millimeter.

The michrom is one-millionth of an ohm, and the megohm is one million ohms. These units are very frequently used in the measurement of very small or very large resistances. The inverse of resistance is known as the conductance of a material. That is, if the resistance of a circuit is $R$ ohms, its conductance is equal to $(1 \div R)$. The unit in which conductance is measured is the mho, and it is the conductance offered by a circuit having a resistance of one ohm.

The resistance of a conductor depends upon its dimensions, the kind of material of which it is composed, and its temperature.

*Resistance of a Conductor Varies Directly as Its Length.*—The resistance of a conductor is directly proportional to its length, the temperature and cross-section of the conductor remaining constant. That is, the resistance increases at the same rate the length of the conductor increases, just as the resistance of a pipe to the flow of water increases as the length of the pipe is increased, all other conditions remaining constant. Hence, if the length of a conductor is increased to four times its original value, the resistance will be four times as much; or if the length is divided into four equal parts, the resistance of each part will be one-fourth of the total resistance.

*Example.*—The resistance of 25 feet of wire is 10 ohms. What is the resistance of 1,000 feet of the same kind of wire?

*Solution.*—Since the resistance of a conductor varies directly as the length, the resistance of 1,000 feet of the wire will be as many times the resistance of 25 feet as 1,000 is times 25. Representing the resistance to be calculated by $R$, we may write

$$R = \frac{1{,}000 \times 10}{25}$$

$$= 400 \text{ ohms}$$

*Resistance of a Conductor Varies Inversely as the Area of Its Cross-Section.*—The resistance of a conductor is inversely proportional to the area of its cross-section, the temperature and the length of the conductor remaining constant, that is, the resistance decreases at the same rate the cross-section increases, all other quantities remaining constant. Hence, if the area of the conductor is reduced to one-fourth its original value, the resistance will be four times as much; or if its area is increased to four times its original value, the resistance will be decreased to one-fourth its previous value.

*Example.*—A conductor .1 square inch in area has a resistance of .75 ohm per foot. What is the resistance per foot of a conductor of the same material .01 square inch in area?

*Solution.*—Since the resistance of a conductor varies inversely as the area, the resistance of one having a cross-section of .01 square inch will be as many times that of one having a cross-section of .1 inch as .1 is times .01. Representing the resistance to be calculated by $R$, we may write

$$R = \frac{.1}{.01} \times .75$$

$$= 7.5 \text{ ohms}$$

*Relation of Resistance to Physical Dimensions.*—From the two previous sections, we know the resistance of a conductor varies directly as its length and inversely as the area of its cross-section. These two relations may be combined with a constant, giving the following equation:

$$R=K\frac{l}{A}$$

The above equation expresses three facts:

(a) The resistance $R$ of a conductor varies directly as the length of the conductor.
(b) The resistance $R$ of a conductor varies inversely as the area of the cross-section of the conductor.
(c) The resistance $R$ of a conductor depends upon the value of a certain constant $K$. The numerical value of the constant $K$ will depend upon the kind of material composing the conductor.

The physical meaning of $K$ may be readily determined by making the length $l$ and the area $A$ of the conductor each equal to unity. $K$, then, is the resistance of a conductor of unit length and having an area of cross-section equal to unity. There will, of course, be as many values of $K$ for a given material as there are units in which to measure the length and the area of the cross-section of the conductor. Only two values of $K$ are in common use, and they are called the *specific* and *mil-foot* resistances.

*Specific Resistance.*—The resistance of a centimeter cube or an inch cube of any material is called the *specific resistance* of the material, and it is usually expressed in michroms at 0 degrees centigrade.

*Mil-Foot Resistance.*—The mil-foot resistance of a material is defined as being the resistance of a volume of the material one foot in length and having a uniform cross-section one circular mil in area. If $l$ is equal to one foot and $A$ is equal to one circular mil, in the above equation, $K$ will be numerically equal to $R$. The mil-foot resistance is really a particular value for the specific resistance, as it is the resistance of a certain volume.

*Circular Mil.*—In the majority of cases, electrical conductors have a circular cross-section, and when we calculate the area of this cross-section by multiplying the square of the diameter by ($\pi \div 4$) the awkward factor .7854 appears. To avoid this factor, a more convenient practical unit of area has been adopted—the *circular mil.* The circular mil is the area of a circle whose diameter is one mil or .001 inch. The square mil is the area of a square whose side is one mil. The area of a circular mil is .7854 times the area of a square mil. Hence, to change from circular mils to square mils you must multiply by .7854, and to change from square mils to circular mils you divide by .7854.

The advantage in the use of the circular mil as a unit of area is that when the diameter of the conductor is given in mils, its circular-mil area may be determined by squaring the diameter. Conversely, the diameter in mils may be determined, when the circular-mil area is given, by taking the square root of the area. The diameter in inches is equal to the diameter in mils divided by one thousand.

*Example.*—Calculate the resistance of a circular conductor 1,000 feet long having a diameter of 102 mils and composed of a material whose mil-foot resistance is 10.8.

*Solution.*—The resistance may be calculated by a direct substitution in the equation

$$R = K\frac{l}{A}$$

which gives

$$\begin{aligned} R &= 10.8\frac{1{,}000}{102^2} \\ &= \frac{10{,}800}{10{,}404} \\ &= 1.03 \text{ ohms} \end{aligned}$$

*Resistance Changes with Temperature.*—A change in the temperature of all substances will cause a change in their electrical resistance. The resistance may decrease with a change in temperature, but in the majority of cases there is an increase in resistance with a rise in temperature. Carbon, perhaps, is the best example of substances whose resistance decreases with an increase in temperature. The resistance of the carbon filament of an incandescent lamp when the lamp is burning is approximately half what it is when the filament is at room temperature or carrying no current.

The change in resistance of materials due to a change in temperature is quite different for different materials, and it is also slightly different for the same material at different temperatures. Alloys, such as manganin, have a very small change in resistance due to a change in temperature, and standard resistances are, as a rule, made from such an alloy, as it is desirable to have their resistance remain as near constant as possible.

*Temperature Coefficient.*—The temperature coefficient of a material is defined as the change in resistance per ohm due to a change in temperature of one degree. Thus, if a copper conductor has a resistance of 1 ohm at 0 degrees centigrade and 1.0042 ohms at 1 degree centigrade, the temperature coefficient of the copper composing the conductor is .0042. The change in resistance of a material per ohm due to a change in temperature of one degree will vary with the initial temperature of the material. Thus, if the resistance of the conductor in the above problem had been 1 ohm at 15 degrees centigrade instead of 0 degrees, the increase in resistance due to a rise in temperature of 1 degree would have been .003801 ohm instead of .0042 ohm. The temperature coefficient of a material is usually represented by the letter *a*, and it is considered positive when a rise in temperature results in an increase in resistance, and negative when a rise in temperature results in a decrease in resistance.

*Calculation of Resistance Due to Change in Temperature.*—If the resistance of a conductor at a given temperature, which we will assume is 0 degrees centigrade, or 32 degrees Fahrenheit, is known, its resistance $R_t$ at some other temperature may be calculated by the use of the temperature coefficient $a$ and the change in temperature $t$, as follows:

$$R_t = R_0 + R_0 at$$

or it is usually written

$$R_t = R_0 \ (1 + at)$$

in which $R_t$ is the value of the resistance at the second temperature; $R_0$ is the resistance at the first temperature; $t$ is the change in temperature in degrees; and $a$ is the temperature coefficient of the material based upon an initial temperature corresponding to the temperature at which $R_0$ is determined. The value of $a$ will also depend upon whether the change in temperature $t$ is measured on the centigrade or the Fahrenheit scale. The value of the temperature coefficients for the Fahrenheit scale are 5/9 of those for the centigrade scale.

The temperature coefficient of a material is usually based upon an initial temperature of 0 degrees centigrade or 32 degrees Fahrenheit. If the initial resistance of the material is not given at 0 degree centigrade it must be calculated by solving for $R_0$ in the above equation and then solving for $R_t$, the resistance at the second temperature. The values of the temperature coefficients for the most common materials are given in Table IV in the Appendix. These values are all based on an initial temperature of zero degrees centigrade. The following example will illustrate the use of the temperature coefficient in calculating the resistance of a conductor at any temperature when its resistance at some other temperature is known.

*Example.*—A copper conductor has a resistance of 22.1 ohms at 25 degrees centigrade, what is its resistance at 50 degrees centigrade?

*Solution.*—Substituting in the equation and solving for $R_o$, the resistance at 0 degrees, we get

$$\begin{aligned} 22.1 &= R_o\,(1 + .0042 \times 25) \\ R_o &= 22.1 \div 1.105 \\ &= 20.0 \text{ ohms} \end{aligned}$$

Now substituting in the same equation and solving for the resistance $R_t$, the resistance at 50 degrees centigrade, we get

$$\begin{aligned} R_t &= 20\,(1 + .0042 \times 50) \\ &= 20 \times 1.21 \\ &= 24.2 \text{ ohms} \end{aligned}$$

*Relative Conductivity of a Material.*—The conductivity of a material is equal to the reciprocal of its specific resistance. The relative conductivity of any material is the relation between the conductivity of the material and that of some other material. Copper is taken as the standard in measuring the conductivity, and copper having a mil-foot resistance of 10.37 at 20 degrees centigrade is assumed to have a conductivity of 100 per cent.

$$\begin{aligned} &\text{per cent relative conductivity} \\ &\qquad = \frac{\text{conductivity of material}}{\text{conductivity of copper}} \times 100 \\ &\qquad = \frac{\text{specific resistance of copper}}{\text{specific resistance of material}} \times 100 \end{aligned}$$

If both materials are not at the same temperatures, a proper correction should be made in the resistance of one due to the difference in their temperatures.

*Wire Gages.*—Circular conductors are usually designated by certain gage numbers rather than by statement of the area of their cross-section. A number of different wire gages have been originated by different manufacturers of wire, but only two are used to any great extent. These are the B. & S. gage originated by the Brown and Sharpe Manufacturing Company, which is the one generally used in this country and commonly called the American gage; and the B. W. G. gage, or Birmingham Wire Gage. There are a number of other gages but they are not used to any great extent. A part of the copper wire table as published by the Bureau of Standards is given in Table III in the Appendix.

*Standard Resistance.*—A standard resistance is one which has been carefully constructed and its value adjusted by the makers so that it may be considered correct. Such resistances are usually constructed of a material having a very low temperature coefficient so as to reduce the effect of a change in temperature to a minimum. The value of their resistance ranges from perhaps .00001 ohm to 1 megohm, and their safe current-carrying capacity from several thousand amperes to only a fractional part of a milli-ampere. Some are so constructed that their temperature may be maintained practically constant, irrespective of the current they are carrying, by means of an oil bath which is kept in circulation and cooled the desired amount by a coil of pipe through which there is a current of water. In ordinary commercial work secondary standards of resistance are employed but their

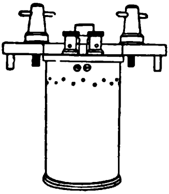


Figure 80.—Standard Resistance.

adjustment and accuracy is usually such that they give sufficiently accurate results for all practical purposes. A standard resistance is shown in Figure 80.

## MEASUREMENT OF RESISTANCE

Practically all the methods employed in measuring resistance depend upon some application of Ohm's law. The method to be used in any case will depend upon the kind of resistance to be measured, that is, whether it is capable of carrying a large or a small current; the accessibility of the resistance; the value of the resistance to be measured; and the accuracy desired. The different methods described in the following sections are, perhaps, the most common ones employed in practice.

*Measuring Resistance by Drop in Potential Method.*—Since Ohm's law holds for any part of an electrical circuit, it follows that the value of a certain resistance can be determined by measuring the difference in electrical pressure between its terminals when the current through the resistance is known. The scheme of connections for measuring resistance by this method is shown in Figure 81, in which *R* is the resistance whose value is to be determined; *V* is a voltmeter of suitable range connected to the terminals of the resistance; *A* is an ammeter for measuring the current in

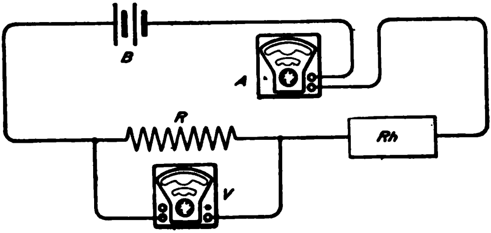


Figure 81.—Drop in Potential Method of Measuring Resistance.

the circuit; *Rh* is the rheostat for adjusting the value of the current; and *B* is a source of electrical energy such as a battery or generator.

Neglecting the current through the voltmeter, the current *I* through the resistance *R* is equal to that indicated by the ammeter *A*, and the value of the resistance *R* is equal to the pressure *E* between its terminals, as indicated by the voltmeter, divided by the current through it, or

$$R = \frac{E}{I}$$

When the resistance of the voltmeter circuit is large in comparison to the value of the resistance *R*, the current through the voltmeter is small in comparison to the current through the resistance *R* and may be neglected. If the resistance of the voltmeter is not large in comparison to the value of *R*, the current through it cannot be neglected, and the value of the voltmeter current $I_v$ must be subtracted from the main line current *I* in order to obtain the true value of the current passing through the resistance *R*, which we will call $I_R$. The current in the voltmeter circuit is equal to the pressure as indicated by the voltmet divided by the resistance of the voltmeter $R_v$, or

$$I_V = \frac{E}{R_V}$$

The current through the resistance *R*, then, is

$$I_R = I - I_V$$

or

$$I_R = I - \frac{E}{R_V}$$

and the value of the unknown resistance will be

$$R = \frac{E}{I_R} = \frac{E}{IR_V - E} \times R_V$$

This method of measuring resistances is best suited to what might be termed current-carrying resistances of low value, such as the armatures of generator and motors, the series coils of wattmeters, etc.

Care should be exercised in making a resistance measurement by this method not to pass too large a current through the object being measured, as you are likely to change its resistance due to a change in temperature resulting from the excessive current. The greater the current, however, without undue heating of the conductor, the greater the voltmeter reading and, as a usual result, the greater the accuracy. When very low resistances are to be measured, a low-reading voltmeter or, better still, a milli-voltmeter should be used.

*Measurement of Resistance by Comparison.*—This method requires no ammeter and the value of the unknown resistance is determined in terms of a known or standard resistance. The scheme of connections for measuring resistance by this method is shown in Figure 82, in which *X* is the resistance whose value is to be determined; *R* is a known or standard resistance; *Rh* is a rheostat for adjusting the current in the circuit; *B* is a source of electrical energy such as a battery or generator; and *V* is a voltmeter of suitable range for measuring the drop in voltage over the known and unknown resistances. The proper connections of the voltmeter for measuring the drops over the two resistances is shown in the figure by the full

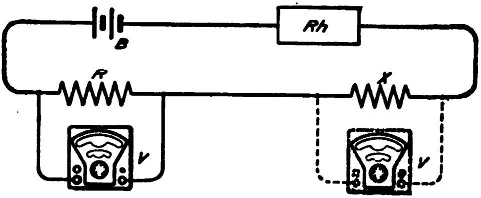


Figure 82.—Measuring Resistance by Comparison.

and dotted lines, and these two readings should both be taken with the same current through each of the resistances. Since the drop across any part of a series circuit bears the same relation to the drop across any other part of the same circuit as exists between the resistances of the two parts, we may write the following simple relation:

$$\frac{\text{drop over } X}{\text{drop over } R} = \frac{\text{resistance of } X}{\text{resistance of } R}$$

or

$$\text{resistance of } X = \frac{\text{drop over } X}{\text{drop over } R} \times \text{resistance of } R$$

Calling the drops over the resistances $X$ and $R$, $E_X$ and $E_R$, we have

$$X = \frac{E_X}{E_R} \times R$$

This method of measuring resistance is, perhaps, best suited to resistances of very low value and capable of carrying quite a large current. The best results are obtained when the values of the known and unknown resistances are as near equal as possible. When the drop in potential over the resistances is very small, a galvanometer may be substituted for the voltmeter, and a greater deflection of its moving system will be produced than in the case of the voltmeter and the per cent of error in observing this large deflection, as a rule, will not be as large as in the case of the smaller deflection. The galvanometer will shunt a smaller part of the current around the resistance to which it is connected than the voltmeter, provided its resistance is greater than the voltmeter, and the current through both $R$ and $X$ will be nearer equal in value.

*Sheldon's Method of Measuring Resistance.*—This method of measuring resistance gives quite accurate results, and the necessary equipment is not expensive or complicated, and the necessary operations are very simple. The scheme of connections for measuring a resistance by this method is shown in Figure 83. The known resistance *R* and the unknown resistance *X* are connected in series by a heavy conductor to the terminals of a circuit capable of supplying a large current. Two resistance boxes are connected in series between the same two points; the boxes are marked *S* and *P* in the figure. One terminal of a sensitive galvanometer *G* is connected between the two resistances *S* and *P;* and the other terminal is connected to the arm of a four-point switch, the four points of the switch being connected to the potential points of the known and unknown resistance as shown in the figure and marked *a, b, c,* and *d.* It is always desirable to have the resistance boxes *S* and *P* identical in construction, and at the start all of the plugs are removed from one of them and a balance obtained by plugging back and forth, keeping the combined resistance in both boxes constant. A balance is obtained with the galvanometer connected in turn to the points *a, b, c,* and *d* and the value of the resistance in the box *S* noted for each adjustment. Calling these resistances $S_a$, $S_b$, $S_c$, and $S_d$, we have the following expression for the value of the unknown resistance *X*:

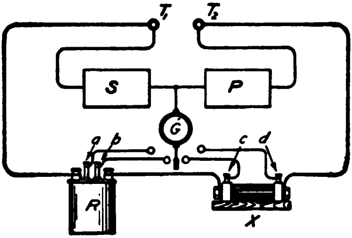


Figure 83.—Sheldon's Method of Measuring Resistance.

$$X = \frac{S_a - S_b}{S_c - S_d} R$$

The best results are obtained with as large a current through the resistances $R$ and $X$ as possible, but it is not necessary to keep this current constant while all the four readings are being taken, unless a change in current results in a change in resistance, in which case it would be best to keep the current as near constant as possible. Reasonable care should be exercised in plugging back and forth between $S$ and $P$ to keep the plugs all tight and in their proper openings. A check should be made after each balance to make sure the combined resistance has not been changed by mistake. This method will be found very satisfactory in determining the resistance of ammeter shunts or comparing their resistances.

*Series-Voltmeter Method of Measuring Resistance.*—The scheme of connections for measuring resistance by this method is shown in Figure 84. The resistance to be measured is connected in series with a direct-reading voltmeter and the combination in turn connected to the terminals of a source of electrical pressure marked $T_1$ and $T_2$ in the figure. A special connection is provided, which shorts the unknown resistance when the connection is closed, as indicated by the dotted lines in the figure. When the resistance $X$ is shorted, the voltmeter reads the total pressure between the terminals $T_1$ and $T_2$, but when the short about the resistance $X$ is removed, the voltmeter indicates only the drop in voltage over itself, the total pressure being divided between the resistance of the

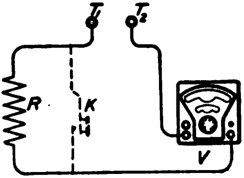


Figure 84.—Series-Voltmeter Method of Measuring Resistance.

voltmeter and the unknown resistance. The drop over the unknown resistance will be equal to the total pressure minus the drop over the voltmeter, or the reading of the voltmeter. Indicating the total pressure by $E$, the drops over the unknown resistance and the voltmeter when they are in series by $E_x$ and $E_v$, respectively, we may write

$$E_x = E - E_v$$

The current through a voltmeter, which we will indicate by $I_v$, is equal to the indication of the instrument in volts divided by its resistance $R_v$, or

$$I_v = \frac{E_v}{R_v}$$

Since the voltmeter and unknown resistance are in series, they are both carrying the same current, and the value of the unknown resistance may be determined by dividing the difference in pressure between its terminals by the current through it, or

$$X = \frac{E_x}{I_x} = \frac{E - E_v}{\frac{E_v}{R_v}} = \frac{E - E_v}{E_v} R_v$$

The above equation gives the value of the resistance $X$ in terms of the pressure between the terminals $T_1$ and $T_2$, which must remain constant while the readings are taken; the drop over the voltmeter $E_v$ when it is in series with the unknown resistance; and the resistance of the voltmeter $R_v$, which is usually marked on the instrument or its containing case. If the pressure between the two terminals $T_1$ and $T_2$ does not remain constant, then a second voltmeter should be connected permanently between them and its reading taken at the same time the voltmeter in series with the unknown resistance is read. The resistance $R_v$ in the above equation is that of the voltmeter in series with the unknown resistance. The best results are obtained when the value of the unknown resistance and that of the voltmeter resistance are as near equal as possible; resistances many times that of the voltmeter may, however, be measured with a fair degree of accuracy.

This method is, in general, serviceable for the measurement of high resistances, such as the insulation of electric light and power circuit that are installed, insulation of trolley lines, dynamos, transformers, etc.

The scheme of connections for the measurement of the resistance of the insulation of a lighting circuit installed in metal conduit is shown in Figure 85.

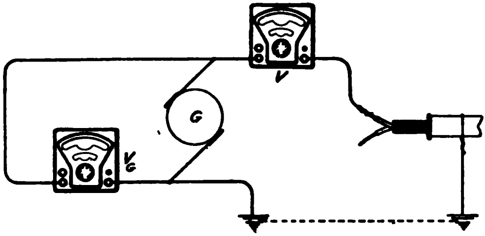


**Figure 85.—Series-Voltmeter Method of Measuring Insulation Resistance.**

A small generator *G* capable of supplying an electrical pressure of about 500 volts is usually used as a source of pressure for testing. One terminal of the generator is connected directly to the conduit, or ground, and the other terminal is connected to the wire whose insulation resistance is to be determined with the voltmeter in circuit. The total pressure generated by the machine *G* will be distributed over the resistance between the wire and the conduit, or ground, and the resistance of the voltmeter. The voltmeter indication will be proportional to the drop across itself, which, subtracted from the total pressure, will give the drop across the unknown resistance $X$. Substituting the values of the total pressure, as read on voltmeter $V_G$ and the drop over $V$, together with the resistance of voltmeter $V$ in the above equation, you can calculate the value of the resistance $X$.

*Example.*—Connections were made, as shown in Figure 85, for testing the insulation resistance of a certain electric-light circuit installed in a metal conduit. The resistance of the voltmeter $V$ connected in series with the resistance to be measured was 50,000 ohms. The voltmeter $V_G$ read 500 volts and the voltmeter $V$, 10 volts. What was the insulation resistance in megohms?

*Solution.*—Substituting the value of the total pressure, the drop over the series voltmeter and its resistance in the equation given above, gives

$$X=\frac{500-10}{10}50{,}000=2{,}450{,}000 \text{ ohms}$$

or

$$=2.45 \text{ megohms}$$

*Direct-Deflection Method of Measuring Resistance.*—In measuring a resistance by this method, a current-measuring instrument, galvanometer, or ammeter, is placed in series with the resistance and the value of the current $I$, a known pressure $E$ volts will produce, is determined. The pressure acting in the circuit divided by the current it produces will give the total resistance of the circuit—combined galvanometer resistance $R_G$, and the unknown resistance $R_X$, neglecting the resistance of the connections. This total resistance, which we will represent by $R$, minus the galvanometer resistance $R_G$, will give the value of the unknown resistance $X$. Writing the above relations in the form of equations, we have

$$R=\frac{E}{I}$$

and

$$X=R-R_G$$

or

$$X=\frac{E}{I}-R_G$$

This method is usually employed in measuring very high resistances, such as the insulation resistance of rubber-covered wires, lead-covered cables, etc., and in such cases a high voltage is required, also a very sensitive galvanometer to measure the extremely small current produced in the high resistance circuit.

In measuring the insulation resistance of insulated conductors without a metal outside covering, they are immersed in a salty solution (it being a better conductor than ordinary water) with at least three feet of each end of the wire out of the solution. One terminal of the testing circuit is connected to the wire itself and the other to a metal plate placed in the solution. The resistance between the wire and the solution is then measured, giving the insulation resistance for the length of the wire immersed in the solution. If the conductor has a metal covering placed outside the insulation, there is no necessity of immersing it in a solution, as the metal covering will serve as one terminal of the insulation resistance.

The scheme of connections for measuring the insulation resistance of a coil of wire is shown in Figure 86, in which $X$ represents a coil of wire immersed in a salty solution; $B$ is a source of electrical pressure; $P$ is a potential rheostat for regulating the pressure acting in the circuit; $V$ is a voltmeter which indicates the pressure acting in the circuit; $G$ is a sensitive galvanometer; $K_1$ is a contact key for short-circuiting the galvanometer; and $K_2$ is a contact key for closing the main circuit. It is imperative that all parts of the circuit be well insulated from each other, in order to obtain good results. The insulation should be removed from the end of the wire for a distance of several inches, and extreme care should be exercised in keeping the surface of the insulation on the ends of the wire as dry as possible so as to reduce the current flowing over the surface of the insulation from the conductor to the solution, which will pass through the galvanometer in addition to the current through the resistance being measured, and thus give a lower apparent insulation resistance than the actual resistance existing between the conductor and the solution. A connection may be made which will conduct any current that leaks over the surface of the insulation around the galvanometer, the only current passing through the galvanometer being that through the insulation between the wire and the solution. Such

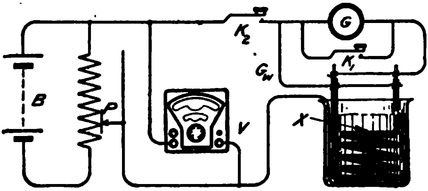


Figure 86.—Scheme of Connections for Measuring the Insulation Resistance of a Coil of Wire.

a connection, which is called a *guard wire*, is shown in the scheme of connections given in Figure 86, and marked $G_w$.

The procedure in making a measurement by this method is as follows: Adjust the potential rheostat $P$ so that the pressure indicated by the voltmeter is very low, then close keys $K_1$ and $K_2$; open key $K_1$ after about one-half minute and take a reading of the galvanometer exactly one minute after the pressure has been applied. Several observations should be made and the average taken. If the deflection of the galvanometer is too large, the pressure should be reduced; if it is not large enough, the pressure should be increased. The current in the circuit is equal to the constant of the galvanometer, as an ammeter, multiplied by the deflection $\theta$, or

$$I = K\theta$$

The pressure as indicated by the voltmeter divided by this current will give the total resistance, which, minus the galvanometer resistance, gives the value of the unknown resistance. If the resistance being measured is very large, the galvanometer resistance may be neglected without any appreciable error.

The insulation resistance of such a circuit is expressed in terms of the megohm-kilometer, megohm-mile, or the megohm thousand feet, meaning the insulation resistance in megohms per kilometer, per mile, or per thousand feet. It varies inversely as the length of the conductor, because with an increase in length there is more surface exposed, resulting in a greater leakage and less resistance. The insulation resistance of large conductors covered with a given thickness of insulation is less than that of small conductors covered to the same thickness, because the area of the leakage path is greater with the large conductors than with the smaller ones.

*Half-Deflection Method of Measuring the Resistance of Galvanometers and Voltmeters.*—If a voltmeter or galvanometer be connected to a constant source of electrical pressure, its resistance may be determined by placing a known resistance in series with it of such a value that the deflection of the instrument is reduced to half its original value, and then noting the value of the series resistance which will be equal to the resistance of the instrument, since the drops over them are equal. The above method requires a variable known resistance, which may not be available, but the ratio of the two deflections of the instrument, with and without the resistance in series, may be taken and the resistance $X$ of the instrument may be calculated from the value of the known resistance $R$ and the values of the two deflections, which we will call $\theta$ and $\theta_R$, by means of the following equation:

$$X = \frac{\theta_R R}{\theta - \theta_R}$$

In the above calculations the deflections are considered proportional to the current through the instrument; but if this is not the case, the value of the scale readings or deflections should be expressed in current or volts and these values used instead of the actual deflections.

*Slide-Wire Bridge.*—Two resistances, which may be equal or unequal, are shown connected in parallel between the points *1* and *2* in Figure 87, and the combination is connected to a source of electrical pressure, such as a battery. The drop in potential across the two branches of the divided circuit is the same regardless of the relation between the two resistances. If the total resistance in each branch is the same, then the drop over a certain number of ohms in the upper branch will be equal to the drop over an equal number of ohms in the lower branch. When the resistances of the two branches are not equal, the above relation does not hold true, but there is a definite relation between the drop over each ohm in the upper branch and that over each ohm in the lower branch. Select some point, such as *3* on the lower branch, and it will always have a potential less than the point *1* and higher than the point *2* when the current is in the direction indicated by the arrows. There is a point on the upper branch whose potential is equal to that of the point *3* and this point may be located by means of a galvanometer, as follows: Connect one terminal of the galvanometer to the point *3* and move the other terminal along the upper branch until a point

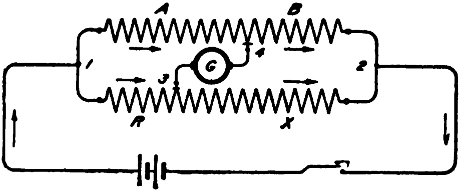


Figure 87.—Fundamental Principle of the Slide-Wire Bridge.

is found which results in no deflection of the galvanometer when its circuit is closed. This point, which is marked *4* in the figure, is at the same potential as the point *3*, as there is no current between them when they are connected by the galvanometer, there being no deflection produced on the galvanometer.

When the point *4* has been located, the drop in potenial over the resistance in the upper branch between the points *1* and *4* is equal to the drop in potential over the resistance in the lower branch between the points *1* and *3*, and the drop in potential across the resistance between the points *4* and *2* is equal to the drop across the resistance between the points *3* and *2*. After a balance is obtained—no deflection of the galvanometer—the resistance between *1* and *3* bears the same relation to the resistance between *1* and *4* as exists between the resistance between the points *3* and *2*, and that between the points *4* and *2*. Calling the resistances between the various points $A$, $B$, $R$, and $X$, as indicated in the figure, we may write the above relation in the form of a simple equation, thus

$$\frac{\text{resistance } A}{\text{resistance } R} = \frac{\text{resistance } B}{\text{resistance } X}$$

For example, suppose the resistance $R$ and the total resistance of the upper branch, $A$ and $B$ combined, are known and the resistance $X$ is unknown. A balance may be obtained, as previously described, and, from the position of the point *4* on the upper branch, the value of the resistance $A$ and $B$ may be determined. The above equation may then be changed to the following form:

$$\text{resistance } X = \frac{\text{resistance } B}{\text{resistance } A} \times \text{resistance } R$$

or

$$X = \frac{B}{A} \times R$$

By substituting the value of the three resistances, *R*, *A*, and *B*, in the above equation, the value of the resistance *X* may be determined. This type of bridge is called a *slide-wire bridge* because one of the branches is usually a piece of resistance wire stretched between the points *1* and *2*.

A commercial form of slide-wire bridge is shown in Figure 88, in which *R* and *X* represent a known and unknown resistance, respectively, and $L_1$ and $L_2$ represent two sections of a resistance wire. A galvanometer *G* has one terminal permanently connected between the junction of the resistances *R* and *X* and the other terminal connected to the resistance wire by a sliding contact as indicated in the figure. A battery *B* supplies current to the divided circuit, and the circuit is opened and closed by means of the key *K*. The galvanometer circuit is open or closed, depending upon whether or not the sliding contact on the resistance wire is pressed down on the wire. A balance is obtained by moving the sliding contact along the resistance wire until there is no deflection of the galvanometer when its circuit is closed or opened, the battery circuit being closed before the galvanometer circuit and opened after the galvanometer circuit. The object of this procedure is to prevent a false balance being obtained due to the

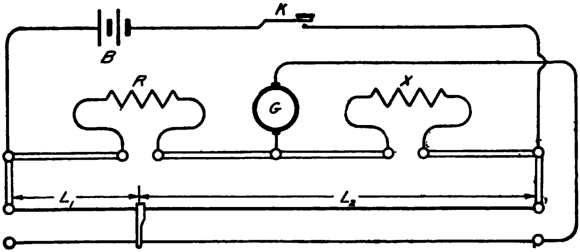


Figure 88.—Simple Slide-Wire Bridge.

effect of any inductance which might be present in the different resistances.

The resistance wire is stretched over a board divided into equal parts and the relation between the resistances of the two parts to the right and the left of the sliding contact, when a balance is obtained, will be exactly the same as the relation between the lengths of the two parts, provided the wire is uniform in cross-section and its material is homogeneous. These two resistances, or lengths of slide wire, are called the *ratio arms,* since their relation to each other gives the relation between the known and the unknown resistances.

*Wheatstone Bridge.*—The form of bridge described in the previous section is not used to any great extent in practice, as its operation is, as a rule, too tedious, and for this reason it is confined almost entirely to laboratory work. The principle of the Wheatstone bridge is the same as that of the slide-wire bridge, differing only in construction and operation. A diagram of a simple form of bridge is shown in Figure 89, and the letters correspond to those in Figure 87. The resistances *A* and *B* are the ratio arms, and they are usually composed of a number of resistance coils whose resistances differ in value by multiples of ten, the lowest being perhaps .1 ohm and the highest

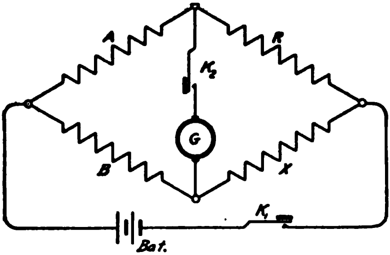


Figure 89.—Wheatstone Bridge.

10,000 ohms. The resistance *R* is called the *rheostat* of the bridge; it is composed of a large number of resistance coils whose resistances usually range in value from .1 ohm to several thousand ohms. The resistance marked *X* corresponds to the unknown resistance. A suitable switching device is usually provided on the top of the case, containing the resistance coil composing the arms *A, B,* and *R,* so that the resistance in any arm may be readily varied from zero to its maximum value. The operation of this bridge differs from the slide-wire bridge in that the relation between the resistances of the ratio arms is fixed and a balance is brought about by adjusting the resistance in the rheostat. Thus, if the resistance of the arms *A* and *B* is 100 ohms each and the resistance in circuit in the rheostat *R* is 125 ohms, then the resistance of *X* is 125 ohms. If the resistance of *A* is 10,000, of *B* is 1,000, and of *R* is 1,250 ohms, respectively, when a balance is obtained, then the resistance of *R* is ten times the resistance of *X* because the resistance of *A* is ten times the resistance of *B*. If the resistance of *A* had been one-tenth that of B, then the resistance of *R* would have been equal to one-tenth that of *X*. The galvanometer and battery connections may be interchanged without interfering with the operation of the bridge, but the best results will be obtained when the galvanometer, if it has a higher resistance than the battery, joins the junction of the two arms of the bridge having the highest resistance to the junction of the two arms having the lowest. Suitable terminals are also provided on the bridge to which the battery, the galvanometer, and the unknown resistance may be connected.

In some of the more modern forms of Wheatstone bridges, both the battery and the galvanometer are built into the bridge, and the only external connections required are those to the unknown resistance. Connecting terminals, however, are usually provided so that an external battery or galvanometer may be used if such be required due to the exhaustion of the battery in the bridge box or if the galvanometer be out of service due to any cause. A very compact form of Wheatstone bridge with the battery and galvanometer self-contained is shown in Figure 90. This particular construction is referred to as the dial pattern, as the resistances of the various arms of the bridge are controlled by turning one or more of the dials on the top of the bridge instead of manipulating plugs, as was done in the first and more common forms of bridges. The various resistance coils composing the different arms of the bridge should all be wound non-inductively, and a wire should be used that has practically no change in resistance due to a change in temperature.

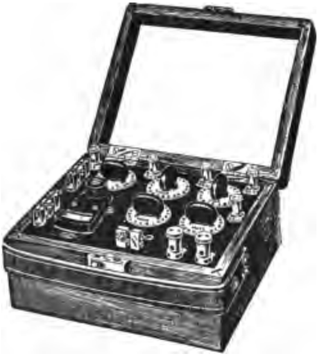


Figure 90.—Commercial Wheatstone Bridge.

The Wheatstone bridge may be used in measuring resistances ranging in value from .1 to 1,000,000 ohms with a fair degree of accuracy. The bridge shown in Figure 90 may be used in making special measurements for the location of grounds and shorts.

*The Ohmmeter.*—An ohmmeter is an instrument for measuring automatically the resistance of a circuit connected to its terminals, by noting the position of a pointer on a dial that is calibrated and marked to read directly in ohms. A commercial form of ohmmeter is shown in Figure 91.

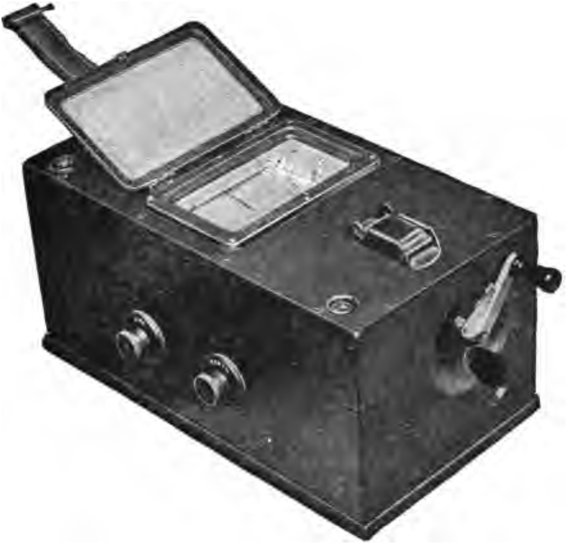


**Figure 91.—Ohmmeter.**

# CHAPTER VII

## MEASUREMENT OF CURRENT AND PRESSURE

*Classification of Instruments.*—Electricity is not a material substance, such as water, and cannot be measured in the same way, since it has no dimensions such as length, breadth, or weight. An electrical current, which is the name given to the flow of electricity in a circuit, is studied and measured by the effects it produces in an electrical circuit. The operation of all electrical measuring instruments depends upon some effect produced by the current and this leads to the classification of instruments into four groups, depending upon the particular effect employed in their operation. These effects are:

(A) Electro-chemical effect
(B) Magnetic effect
(C) Heating effect
(D) Electrostatic effect

The electrostatic effect is not an effect of a current primarily but of electrical pressure.

In addition to the above classification all instruments may be divided into the following groups:

(a) Instruments suitable for direct-current measurements only.
(b) Instruments suitable for alternating-current measurements only.
(c) Instruments suitable for both direct- and alternating-current measurements.

In the following discussion of instruments, they will be grouped according to the effect upon which their operation depends and their adaptability to measurements in direct- or alternating-current circuits will be pointed out at the same time. No attempt will be made to describe all of the various forms and makes of instruments on the market at the present time, but the description will be confined, in almost every case, to those types that are in common use.

*Distinction Between Galvanometers, Ammeters, and Voltmeters.*—A *galvanometer* is an instrument used in detecting the presence of a current in a circuit, or for measuring the value of the current. Galvanometers are, as a rule, quite sensitive and generally used in detecting or measuring very small currents. The resistance of galvanometers varies from a fractional part of an ohm to several thousand ohms, depending upon the particular use that is made of the instrument.

An *ammeter* is an instrument used in measuring the value of the current in a circuit, and for that reason ammeters will always be connected in series with that part of the circuit in which it is desired to measure the value of the current. The scales of the instruments are usually marked to read directly in amperes or milli-amperes. In Figure 92, the ammeter $A$ is connected in series with the battery $B$ and the two resistances $R_1$ and $R_2$, which are connected in parallel. The ammeter in this case reads the total current in the main circuit but it does not give the current in either of the resistances $R_1$ or $R_2$. If, however, the ammeter be connected, as indicated in Figure 93, it will read the value of the current in the resistance $R_1$.

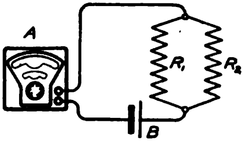


Figure 92.—Ammeter Indicates Total Current.

By changing the ammeter to the branch containing the resistance $R_2$, the current in the second branch can likewise be determined.

The resistance of an ammeter should be as low as possible for the following reasons: Since an ammeter will always be connected in series in the circuit, there will be a difference in electrical pressure between its terminals when there is a current flowing through it which, at any instant, is numerically equal to the product of the current and the resistance of the instrument. This drop in pressure between the terminals of the ammeter should be small in order that the power required to operate the instrument be low in value, it being equal to the product of the current through the ammeter and the difference in pressure between the ammeter terminals.

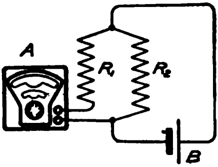


Figure 93.—Ammeter Indicates Current in One Branch.

If the ammeter in Figure 92 had a resistance whose value was something near the combined resistance of the two coils $R_1$ and $R_2$ in parallel, practically half the output of the battery would be consumed in the ammeter and would represent a loss. If, on the other hand, the resistance of the ammeter was very low in comparison to the combined resistance of the two coils, a very small part of the total output of the battery would be consumed in it.

The division of the total current between the two branches of the circuit shown in Figure 93 would be quite different with an ammeter having a high resistance from what it would be with one having a low resistance. The lower the resistance of the ammeter, the less change there will be in the relation between the currents in the two branches when the ammeter is connected in either of them.

A *voltmeter* is an instrument for measuring the difference in electrical pressure between any two points to which its terminals may be connected. Thus, if it is desired to know the difference in electrical pressure between the terminals of any part of an electrical circuit, such as the difference in pressure between the terminals of a lamp that is connected to some source of electrical energy, as indicated in Figure 94, the voltmeter terminals are connected to the terminals of the lamp and the indication of the voltmeter, when its scale is properly calibrated, is a measure of the pressure acting on the lamp. Voltmeters operate on the same principle as the ammeter and the galvanometer, that is, their indication depends upon the value of the current passing through them. The voltmeter indication varies as the pressure between its terminals because the current through it varies with this difference in pressure, the resistance of the instrument remaining constant.

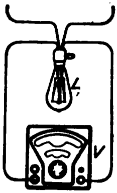


**Figure 94.—Voltmeter Indicates Pressure Acting on Lamp.**

The resistance of a voltmeter should be as large as possible for the following reasons: The loss of power in the voltmeter is equal to the product of the current through it and the difference in pressure between its terminals, and since it is to indicate the difference in pressure, the only way that this loss can be reduced is to increase the resistance of the instrument, which results in a smaller current.

If the voltmeter *V*, Figure 94, had a rèsistance equal to that of the lamp *L*, then the power required to operate the voltmeter would be equal to that required to operate the lamp, since there would be the same current through each and the same difference in pressure between their terminals. This condition would result in a considerable loss and the only way it can be reduced is to increase the resistance of the voltmeter, which results in it taking less current. An ammeter connected in the circuit would indicate the combined current in the lamp and the voltmeter circuits. Hence, in order to get the current in the lamp circuit, the value of the current in the voltmeter circuit should be subtracted from the value of the total current. If the resistance of the voltmeter be large in comparison to that of the lamp, the current in it will be small in comparison to that in the lamp circuit, and the current indicated by the ammeter will be practically equal to that in the lamp. From the above discussion it is seen that an ammeter should always have as low a resistance as possible and a voltmeter as high a resistance as possible in order that the power required to operate them be small.

Care should always be exercised in the use of ammeters and voltmeters to be sure that you are connecting them in the proper way, and always make sure that the current in the circuit or the value of the pressure does not exceed the rated capacity of the instruments.

*Ammeter Shunts.*—In the construction of certain types of ammeters, it is impossible to send the total current through the part of the ammeter in which the effect of the current is produced, as the mechanical construction of such parts usually limits their current-carrying capacity to rather low values. The current capacity of such ammeters may be increased by connecting them in parallel with a definite resistance in the case of a direct-current measurement, or to the terminals of a current transformer, in the case of alternating-current measurement. The resistance connected in parallel with the ammeter is called an *ammeter shunt,* and their resistance is usually adjusted so that the ammeter carries a known fractional part of the total current. The scale of the ammeter may be marked to read directly in amperes without any shunt, and the scale reading multiplied by a constant when the shunt is used. The value of the constant will depend upon the fractional part of the total current the ammeter is carrying. Thus, if the resistance of the ammeter is four times that of the shunt, it will carry one-fifth of the total current and its readings should be multiplied by five in order to obtain the value of the total current. Very frequently the shunt is mounted in the same case as the ammeter, and to all appearances there is only one circuit through the instrument. The scale of such instruments are marked to read the total amount flowing from one terminal of the instrument to the other. When the capacity of the instrument is several hundred amperes or more, the shunt is constructed separately. This type of construction has the advantage of permitting the shunt being connected in the circuit

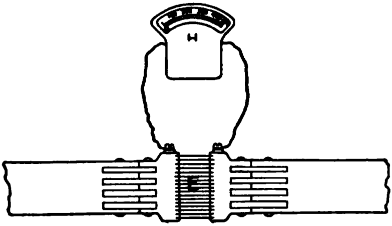


Figure 95.—Switchboard Ammeter with Shunt.

at the desired point and the ammeter then located at some other point, as the ammeter itself carries a small part of the total current and requires small leads. Such an arrangement is usually used on switchboards, and an instrument of this type is shown in Figure 95. One ammeter may be used in combination with several shunts, it being connected by means of a special switch to the different shunts one at a time. The variation in contact resistance at the switch should be very small and the resistance of the ammeter itself should be as large as possible in comparison to that of the shunt in order to reduce the error due to a variation in the switch contact resistance.

*Voltmeter Multipliers.*—The range of any voltmeter, with the exception of electrostatic voltmeters, may be increased by connecting a resistance in series with the instrument. Thus, if a resistance equal to the resistance of the voltmeter be connected in series with the instrument, the instrument will indicate only half the pressure between the two points to which the terminals of the combination are connected. A resistance used in increasing the range of a voltmeter is called a *multiplier*, and their resistance is usually a whole number of times the resistance of the voltmeter. If a resistance equal to nine times the voltmeter resistance be connected in series with the voltmeter, then the drop-over, the voltmeter is one-ninth that over the series resistance, and the voltmeter indication must be multiplied by ten in order to obtain the value of the total pressure acting on the combination. Any resistance may be used as a multiplier provided its value is known so that the constant by which the voltmeter indication must be multiplied may be calculated. A voltage in excess of the range of a given voltmeter may be determined by connecting two voltmeters in series and adding their indication. If the two voltmeters each have the same resistance, they will indicate the same when connected in series; if their resistances are unequal their indications will be proportional to their respective resistances.

The range of an alternating-current voltmeter may be increased by placing a non-inductive resistance in series with it, just as in the case of the direct-current voltmeter. The usual method, however, of increasing the range of an alternating-current voltmeter is by means of a potential transformer.

*Transformer.*—A transformer, in brief, consists of two coils wound about a common magnetic circuit, and it is used in transforming energy from an alternating-current circuit in which one of the coils may be connected to a second circuit in which the second coil is connected. The first coil, or the one connected to the source of electrical energy, is called the *primary winding*, and the second coil is called the *secondary winding*. The relation of the current and the voltage for the primary circuit may be quite different than the relation of these two quantities for the secondary circuit, and transformers are, as a result of this difference, divided into three groups. These groups are:

(a) Potential transformer
(b) Current transformer
(c) Constant-current transformer

A *potential transformer* is one in which the voltage between the terminals of the secondary circuit always bears a definite relation to the voltage impressed upon the primary winding.

A *current transformer* is one in which the current in the secondary winding always bears a definite relation to the current in the primary winding.

A *constant-current transformer* is one in which the current in the secondary winding remains practically constant independent of the value of the current in the primary winding.

Since the current in the secondary winding of a current-transformer always bears a definite relation to the current in the primary winding, this transformer may be used in combination with a low-reading alternating-current ammeter in measuring the current in an alternating-current circuit, as the ammeter shunt is used in the direct-current circuit. The transformer is so constructed that the current in the secondary winding is a definite fractional part of the current in the primary winding, and the indication of an ammeter connected in the secondary circuit must be multiplied by a constant whose value depends upon the relation between primary and secondary currents, unless the scale of the ammeter is marked to read directly in amperes in the primary circuit.

*Example.*—The ratio of primary to secondary current in a certain current transformer is 50 to 1. What current is there in the primary winding when an ammeter connected in the secondary winding indicates 5 amperes?

*Solution.*—Since the ratio of primary to secondary current is 50, then the primary current is 50 times the secondary current, or

$$\begin{aligned} I_p &= 50 \times I_s \\ &= 50 \times 5 = 250 \text{ amperes} \end{aligned}$$

One ammeter may be used with any number of current transformers, but the secondary winding of the different transformers must never be opened while there is a current in the primary winding, as the transformer will either be injured or destroyed. A special switch must be provided which will short all

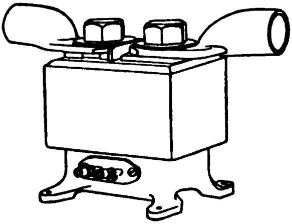


Figure 96.—Switchboard Type Current Transformer.

the secondaries except the one to which the ammeter is connected. It is always advisable to use as many ammeters as current transformers and the above trouble is reduced to a minimum. Two forms of current transformers are shown in Figures 96 and 97. The transformer shown in Figure 96 is a switchboard type of current transformer, and its primary is so constructed that it may be connected permanently in the main circuit. The transformer shown in Figure 97 is a portable type of current transformer, and it is composed of a single coil which is placed about a specially constructed iron ring which encircles the conductor, in which the current is to be measured. The coil itself forms the secondary winding and the conductor the primary winding.

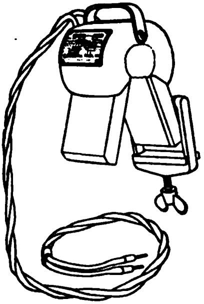


Figure 97.—Portable Type Current Transformer.

The potential transformer serves the same purpose in the alternating-current circuit that the multiplier does in the direct-current circuit. The potential transformers are usually constructed so that their secondary voltage bears a definite relation to the primary voltage, and the readings of a voltmeter connected to the secondary terminals must always be multiplied by a constant whose value depends upon the relation of primary and secondary voltages, in order to obtain the value of the voltage between the terminals of the primary winding, unless the scale of the instrument is marked to read directly in volts.

*Example.*—A potential transformer is so wound that the secondary voltage is one-tenth of the primary voltage. What pressure exists between the terminals of the primary winding when a voltmeter connected to the terminals of the secondary circuit indicates 110 volts?

*Solution.*—Since the ratio of primary voltage to secondary voltage is 10, then the primary voltage will be 10 times the secondary, or

$$\begin{aligned} E_p &= 10 \times E_s \\ &= 10 \times 110 = 1{,}100 \text{ volts} \end{aligned}$$

A single voltmeter may be used in combination with several potential transformers in determining the voltage of different circuits, a special switch being provided by means of which the voltmeter connections may be shifted from one secondary winding to the other.

A great advantage in the use of current and potential transformers is due to the fact that their primary windings, which may be connected to a very high voltage circuit, are insulated from the secondary windings, and the likelihood of an operator or attendant in the power house coming into contact with the high-voltage circuit is thus greatly reduced.

The relation of primary to secondary current in the current transformer is not constant for all values of primary current. This relation also depends upon the resistance connected between the terminals of the secondary winding, and for these reasons the ratio of primary to secondary current is determined for all values of primary current the winding is supposed to carry with a definite impedance, or a particular ammeter connected in the secondary winding.

The relation of primary to secondary voltages, in the case of the potential transformer, will change with a change in the value of the current the secondary winding of the transformer must supply. The ratio of voltage and current transformation is usually assumed constant for all practical purposes and the exact values are used only in very accurate measurements.

Current and potential transformers may be used in operating a number of instruments on a switchboard at the same time, such as voltmeters, ammeters, wattmeters, power-factor meters, frequency meters, etc. A form of potential transformer for pressures up to 15,000 volts is shown in Figure 98.

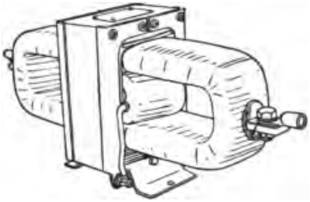


**Figure 98.—Potential Transformer.**

## INSTRUMENTS WHOSE OPERATION DEPENDS UPON THE ELECTRO-CHEMICAL EFFECT OF A CURRENT

*Electrolysis.*—If two conducting plates, such as platinum, be immersed in acidulated water and a current of electricity be passed through the solution from one plate to the other, the water will be decomposed into its two constituents, oxygen and hydrogen. Such a combination of plates and solution constitutes what is called an *electroylic cell,* and the process of decomposing the liquid is called *electrolysis.* The solution through which the electricity is being conducted is called the *electrolyte,* and the two plates immersed in the solution are called the *electrodes.* The plate by which the electricity enters the solution is called the *positive electrode,* or *anode,* and the plate by which the electricity leaves the solution is called the *negative electrode,* or *cathode.* The parts into which the electrolyte is decomposed are called *ions,* and the ion liberated at the positive electrode is called the *anion,* while the ion liberated at the negative electrode is called the *cathion.* The vessel, plates, and other apparatus used in electrolysis constitute what is termed a *voltameter,* when such apparatus is used to measure a quantity of electricity or the average current in a circuit.

When water is decomposed by an electric current, hydrogen appears at the negative plate, or it is the cathion, and oxygen appears at the positive plate, or it is the anion. A simple electrolyte cell is shown in Figure 99.

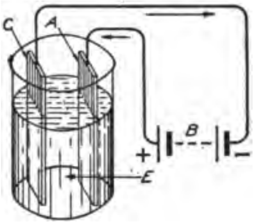


Figure 99.—Simple Electrolytic Cell.

*Coulomb and Ampere.*—The *coulomb* is the unit of quantity of electricity, and it is that quantity which will, under definite fixed conditions, deposit .001118 gram of silver from a solution of nitrate of silver in water. If this quantity of silver is deposited in exactly one second and the rate at which the electricity is transferred is the same at each instant, there is said to be a current of one ampere in the circuit.

The weight of a metal deposited by a coulomb of electricity, when caused to pass through a solution of a salt of the metal and water, is called the electrochemical equivalent of the metal. The values of the electro-chemical equivalent for some of the common metals are as follows:

| | | | |
|---|---|---|---|
| silver | .001118 | zinc | .000338 |
| copper | .000329 | lead | .001071 |
| nickel | .000304 | | |

The voltmeter may be used as a quantity-measuring instrument when the constant of the metal in the electrolyte is known, and the value of the quantity in coulombs will be equal to the increase in weight of the cathode in grams, divided by the value of the constant.

*Examples.*—1. The plates forming the cathode of an electrolytic cell, having an electrolyte of zinc sulphate, are increased in weight 1.014 grams. What quantity of electricity passed through the cell?

*Solution.*—The quantity of electricity in coulombs is equal to the increase in weight divided by the electrochemical equivalent of the zinc, or

$$\begin{aligned} Q &= 1.014 \div .000338 \\ &= 3{,}000 \text{ coulombs} \end{aligned}$$

2. Assuming that the quantity of electricity in the above example passed through the electrolyte at a uniform rate for a period of 2.5 minutes, what was the current in the circuit in amperes?

*Solution.*—Since the current is equal to the quantity in coulombs divided by the time in seconds, we have

$$\begin{aligned} I &= 3{,}000 \div (2.5 \times 60) \\ &= 20 \text{ amperes} \end{aligned}$$

*Adaptability of Voltameters.*—The voltameter is primarily a quantity-measuring instrument and it will measure the current only when the rate of flow of electricity is constant and, as a result, it is not suitable for ordinary work. Determinations of the value of a current made by the instrument are very accurate; and when properly made, they are used as primary standards in checking up the indications of other current-measuring instruments.

The voltameter is not suitable for pressure measurements, since its resistance is not constant and, as a result, the current through it would not always be proportional to the pressure impressed upon the circuit of which the voltameter is a part. The voltameter would give the average value of the pressure for a given time, if its own resistance were to remain constant, rather than the pressure at some particular instant.

Voltameters cannot be used in alternating-current measurements, since there would be a reversal of chemical action each time there was a change in the direction of the current, and if the same quantity of electricity passed through the instrument in one direction as passed through it in the other, there would be no metallic deposit on either of the sets of plates.

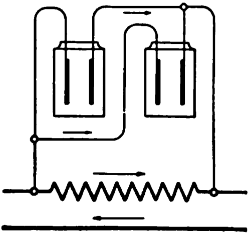


Figure 100.—Connections of Electrolytic Meter.

The old Edison chemical meter and the Wright electrolytic meter, which is used quite extensively in Europe, depend upon the electrolytic action of the current. These meters are connected in parallel with a shunt, as shown in Figure 100, and only a fractional part of the current in the main circuit passes through them. Meters of this type are also used as watt-hour meters, in which case they are usually calibrated to read directly in kilowatt-hours at some assumed voltage.

## INSTRUMENTS WHOSE OPERATION DEPENDS UPON THE MAGNETIC EFFECT OF A CURRENT

*Magnetic Effect of a Current.*—The magnetic effect of a current was discussed in Chapter II and it is only necessary to bear in mind the following facts:

(a) There is a magnetic field about a conductor in which there is a current due to the current in the conductor.

(b) The direction of the magnetic field, due to the current in the conductor, will depend upon the direction of the current.

(c) The strength or intensity of the magnetic field at any point about a conductor carrying a current will vary with the value of the current in the conductor and the distance the point is from the conductor.

(d) If a conductor, in which there is a current, be placed in a magnetic field in any position other than parallel to the field, it will be acted upon by a force which tends to move it in a path perpendicular to the direction of the field and whose value depends upon the strength of the magnetic field, the value of the current, the length of the conductor in the field, and the angle the axis of the conductor makes with the direction of the magnetic field.

Instruments whose operation depend upon the magnetic effect of a current in a conductor may be classified as follows:

(A) Those in which permanent magnets are used, they being acted upon by the magnetic field produced by the current. Either the magnet or the conductor carrying the current may form the moving part.

(B) Those having soft iron or other metal parts which are moved, due to the magnetic effect produced by the current in the conductor.

(C) Those in which no iron is used, but having two coils, one of which is movable. This coil is moved, due to a magnetic force that is exerted between them when there is a current in both coils.

### CLASS "A" INSTRUMENTS

*The Tangent Galvanometer.*—If a magnetic needle be supported in the center of a coil, as shown in Figure 101, there will be a force tending to turn the needle from its position of rest in the earth's magnetic field when there is a current in the coil, provided the plane of the coil is not perpendicular to the direction of the earth's field. This force will increase with an increase in current in the coil and, as a result, the needle will be deflected more and more as the current is increased until it occupies a position almost perpendicular to the plane of the coil. If a suitable scale be constructed and mounted directly beneath the needle, so that the needle, or a pointer attached to the needle, will move over the scale, the deflection of the needle from the position of rest, due to a certain current, can be determined.

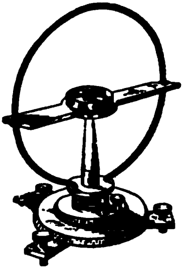


Figure 101.—Tangent Galvanometer.

The force which tends to return the needle to its initial position is due to the magnetic field of the earth. In using the instrument the coil carrying the current should be placed with its plane parallel to the plane of the needle and allowed to remain in this position while all readings are being taken. Such an instrument is usually called a *tangent galvanometer*, or *ammeter*, because the current in the coil is equal to some constant times the tangent of the angle through which the needle moves. The indications of an instrument of this kind are disturbed by the presence of magnetic fields other than that of the earth, or the current in the coil, and, as a result, its operation is not very satisfactory except under ideal conditions.

*D'Arsonval Instrument.*—In the D'Arsonval type of instrument, the permanent magnet is the stationary portion, and the conductor carrying the current forms the moving part of the instrument. The instrument is named after a French scientist, who first put it into a useful form. The conductor carrying the current to be measured is bent into the form of a coil and may be either suspended or supported in the magnetic field

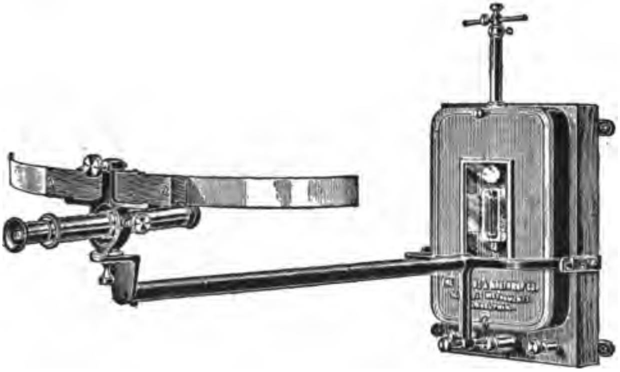


Figure 102.—D'Arsonval Galvanometer.

of one or more permanent magnets. In the most sensitive forms of the instrument the coil is usually suspended by a conducting thread, such as phosphor-bronze or plated quartz fiber. The electricity is conducted to and from the coil by means of this support and another electrical connection at the bottom of the coil—which may consist of a second fiber of the same material as the upper one—or it may be a fine wire coiled into a spiral, or a wire may dip into a cup of mercury.

A D'Arsonval galvanometer, complete with telescope and scale, is shown in Figure 102. The deflections of the coil are measured by means of the telescope and scale, together with a small mirror mounted on the moving system of the instrument. The image of the scale in the mirror may be read by means of the telescope and, as the mirror turns, the part of the scale visible through the telescope changes. The deflection of the moving system, as read on a curved scale, will always be twice the actual deflection. If a straight scale is used, the readings on the scale, due to a given movement of the coil, increase in value with an increase in deflection, due to the fact that the ends of the scale are farther from the mirror than the center.

*The Ballistic Galvanometer.*—The ballistic galvanometer is a form of galvanometer having a comparatively heavy moving system, which results in the galvanometer coil not moving from its zero position, or position of rest, immediately after it is connected to a source of electrical pressure. As a result of this property of the galvanometer, it is possible to almost completely charge or discharge a condenser through it before the moving system has moved appreciably from its zero position. In such a galvanometer, the deflection is produced in a manner similar to the action of a rifle bullet striking a suspended weight. The energy of the bullet is imparted to the weight which causes it to deflect and come to rest at an extreme position. When the weight comes to rest in its extreme position, the energy imparted to it has all been used in overcoming the action of gravity, as the weight is now in a higher position than when at rest. In the ballistic galvanometer the energy of the charge is imparted to the moving system and causes it to revolve against the action of a spring or suspension until the energy of the quantity is all stored in the spring or suspension. The moving system does not remain in this extreme position but immediately starts to return to its zero position and in doing so the energy of the spring or suspension is imparted to it, neglecting certain losses, and it is deflected to an extreme position in the opposite direction. The moving system would continue to move back and forth between these two positions for an indefinite period if it were not for the action of certain forces, called *damping forces,* the principal ones of which are the friction of the air and the action of the induced currents produced by the relative movement of the coil and magnetic field due to the permanent magnets. As a result of these damping forces, the actual deflection produced by a given quantity of electricity is not as great as would occur if there were no damping forces present. The observed deflections must be multiplied by a factor, called the *damping factor,* which is always greater than unity, in order to get the theoretical deflection. The value of the damping factor is not constant but depends upon the resistance in series with the instrument when the observed deflection is produced. The experimental determination of the damping factor is explained in Chapter XI.

*D'Arsonval Ammeters and Voltmeters.*—The moving coil in these instruments is mounted on pointed pivots which are extremely hard and rest in agate jewels, which results in a very small frictional resistance to the movement of the coils. A light pointer is attached to the coil and so arranged that it moves over a suitable scale properly graduated and lettered so that the indication of the instrument may be easily determined.

A view of the moving system of an instrument of this type is shown in Figure 103, with a portion of the instrument removed so as to show the construction. The permanent magnet is of the horseshoe type and has two pole pieces fastened to its ends by means of heavy screws. The inner surface of each of these pole pieces is cut away so it forms an arc, the center of which is midway between the two inner surfaces of the ends of the magnet. A soft-iron cylinder is mounted between the two pole pieces and serves to improve the magnetic circuit by reducing its reluctance. The magnetic lines cross the air gap between the pole pieces and the soft-iron core perpendicular to their surfaces; and since the length of the air gap is practically constant, the magnetic field will be uniform for all positions in the air gap except near the edge of the pole pieces. The coil carrying the current to be measured is mounted so that it can turn about the soft-iron cylinder. The force tending to return the coil to its zero position is supplied by two springs, one above and the other below the coil and so arranged that the one winds up and the other unwinds, due to

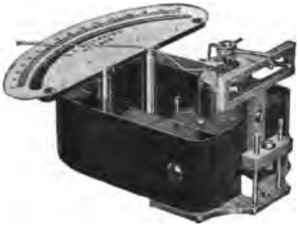


Figure 103.—View of D'Arsonval Instrument.

any movement of the coil. The electrical connections to the coil are made through these two springs.

Practically the same moving system is employed in the construction of ammeters and voltmeters. In the ammeters, except those used in measuring very small currents, a low resistance, capable of carrying the current the instrument is supposed to measure, is connected between the binding posts on the instrument case; and the moving system is connected in parallel with this resistance. Binding posts are provided of ample size to carry the current. A Weston portable ammeter is shown in Figure 104.

In the voltmeter, a resistance is connected in series with the moving system between the terminals of the instrument. The value of the resistance to be placed in series with any moving system will depend upon the current required to give a full-scale deflection and on the voltage that a full-scale deflection is to indicate. Thus, if a certain moving system requires .01 ampere to produce a full-scale deflection and it is desired that the full deflection correspond to a pressure of 150 volts, it will be necessary to have a total resistance between the terminals of the instrument of (150 ÷ .01), or 15,000 ohms.

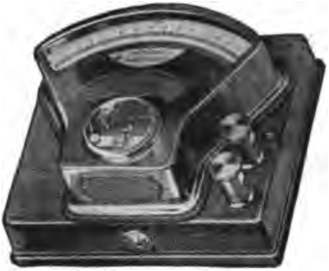


Figure 104.—Weston Ammeter.

Voltmeters are usually constructed with more than one combination of terminals, and the resistance in circuit with the different combinations are, as a rule, whole multiples of each other. If in the above case the resistance of a second circuit through the moving system was 300 ohms, a full-scale deflection would be produced by a pressure of 3 volts. The resistances in series with the moving system are mounted in the same case as the system itself, except in the case of very high voltages, when an outside resistance, called a *multiplier*, is connected in series with the instrument. One terminal of all the circuits through the instrument are usually connected to a common binding post and the remaining terminals are connected to individual posts. Independent series resistances are usually used in each circuit so that the adjustment of one resistance will not interfere with that of any of the others. A contact key is usually placed in series with the common binding post, which serves to open or to close the voltmeter circuit. A Weston portable voltmeter is shown in Figure 105.

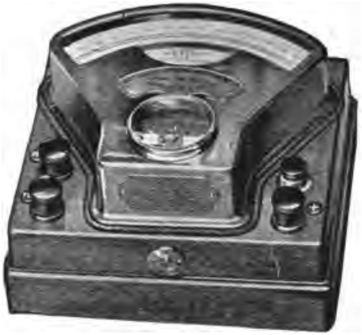


**Figure 105.—Weston Voltmeter.**

A *milli-voltmeter* is an instrument used in measuring very low pressures. Its resistance is very low in comparison to the ordinary voltmeter, in order that the required deflecting current may be produced by the very low pressure between its terminals. A small change in the resistance between the terminals of such an instrument will make quite a large change in the deflection produced by a given pressure, and, as a result, no contact key is used due to its variable resistance of contact, and special leads are provided so that the same resistance will always be in series with the moving system. When the instrument is calibrated with a given set of leads, it indicates the pressure between the ends of the leads and not between the binding posts of the instrument.

Milli-voltmeters are used in combination with known resistances in measuring current. The milli-voltmeter is connected between the pressure terminals of the resistance, and its indication is the drop over the resistance. This drop divided by the value of the resistance will give the value of the current. Any number of current ranges may be had by using known resistances of different values. These resistances should have sufficient current-carrying capacity to take care of the maximum current of the circuit.

*Adaptability of Instruments with Permanent Magnets.*—D'Arsonval instruments are used very extensively in practice and there are a number of makes on the market which give satisfactory results under various conditions of operation. They are "dead-beat"; that is, the coil, or moving system, swings immediately to its proper position without swinging back and forth, as in many of the other types. The scales of instruments of this type are practically uniform throughout their entire length. They can, however, be used only in direct-current measurements, because the direction of the deflection is dependent upon the direction of the current through them.

*Thomson Astatic Instruments for Direct-Current Switchboards.*—The Thomson astatic instruments are a modification of the D'Arsonval type of instrument.

The fields of these instruments are electromagnets wound for any specified voltage and provided with binding posts separate from the current posts of the instrument. The moving coils are mounted upon an aluminum disk and are located in a magnetic field which is parallel to the shaft and astatically arranged. Two small pieces of magnetic metal are rigidly mounted on the shaft, and the astatic components of the magnetic field, which are perpendicular to the shaft, tend to keep the pieces of magnetic metal in their initial positions. When current passes through the coils of the moving element, the lines of force parallel to the shaft produce a torque which tends to turn the shaft and cause the needle to travel across the scale. This action is, of course, opposed by the magnetic field at right angles to the shaft acting on the two pieces of magnetic metal. These instruments have no controlling springs. The current is conducted to and from the movable element by two small spirals

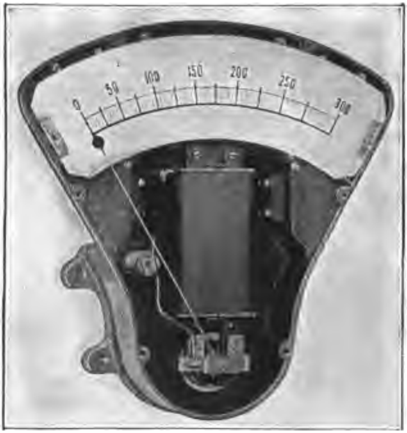


Figure 106.—Thomson Astatic Ammeter, Cover Removed.

made of silver alloy and they exert no force as springs. The actuating and restraining forces are dependent upon the same electromagnets. However the strength of the magnets may change, the relative distribution of the field cannot change; and consequently, the accuracy of the instrument cannot be affected. Therefore the two variable elements common in many direct-current instruments are eliminated and the calibration of the instrument is practically constant.

The effects of external field are eliminated by the astatic arrangement of the fields and moving parts. A field which tends to increase the torque on one side of the moving element diminishes it to a corresponding degree on the other side. These instruments are made as ammeters and voltmeters for direct-current switchboard use only. A front view of an instrument of this type, with the cover removed, is shown in Figure 106.

### CLASS "B" INSTRUMENTS

*Plunger Type Ammeters and Voltmeters.*—A diagrammatic representation of the construction of a plunger type ammeter or voltmeter is given in Figure 107. The coil *C* carries the current to be measured, and there is a magnetic force exerted on the rod of iron *R*, when there is a current in the coil, which tends to draw the rod into the coil and thus cause the

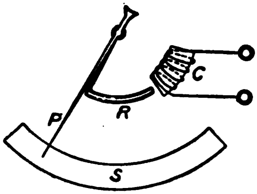


Figure 107.—Principle of Plunger Type Instrument.

pointer *P* to move over the scale *S*. Gravity, in this case, is the controlling force against which the magnetic action of the coil acts. The rod *R* is composed of very soft iron, or laminated iron, and it becomes a magnet due to the action of the current in the coil *C* and, as a result, is drawn inside the coil. The distance the rod moves, due to a given current, will depend upon the value of the controlling force and the ampere turns on the coil. When the instrument is to be used as an ammeter, the coil is wound with a few turns of large wire and the necessary ampere-turns are produced by a large current through a few turns. In the voltmeter the coil is wound with a large number of turns of small wire and the necessary ampere-turns are produced by a small current through a large number of turns. This construction results in the winding of the ammeter having a low resistance and the winding of the voltmeter a high resistance, which is a desirable condition in order that the power required to operate them be low.

*Magnet-Vane Ammeters and Voltmeters.*—Instruments of this type operate on the principle that a piece of soft iron, when placed in a magnetic field and free to move, will always move into such a position that it will conduct the maximum number of lines of force, or it will tend to move into the strongest part of the field and with its longest dimension parallel to the direction of the field. A diagrammatic representation of an instrument of this type is shown in Figure 108. The coil *C* carries the current to be measured, and this current produces a magnetic field within the coil which will be approximately perpendicular to the plane of the coil. A small piece of soft iron *V*, called the *vane,* is mounted on a shaft which is supported in jewel bearings with its axis parallel to the axis of the coil. The shaft carrying the vane is not in the center of the coil, which results in the distance the vane is from the inner edge of the coil changing as the shaft rotates, the distance decreasing with an increase in the deflection of the moving system. A pointer $P$ is attached to the shaft and moves over the scale $S$, and a small spiral spring $S_1$ supplies the controlling force which keeps the pointer at zero when there is no current in the coil. The magnetic field inside the coil, due to a given current, increases in strength as you pass from the center toward the inner edge and, as a result, the vane will move so that the distance between it and the inner edge of the coil is as small as possible, if it were not for the restoring force supplied by the coiled spring. The tendency of this vane to rotate increases with an increase of current in the coil and, as a result, the pointer attached to the moving system moves over the scale with an increase in current until there is a balance between the restoring force of the spring and the deflecting force, due to the action of the magnetic field upon the vane.

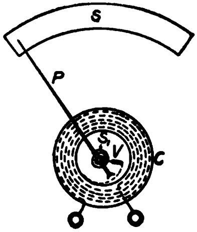


**Figure 108.—Principle of Magnet Vane Instrument.**

In some instruments of this type there is both a stationary and a movable vane. They both become magnetized due to the current in the coil, and, since the polarity of their magnetic poles at the upper and the lower ends will be the same, they will repel each other. If the stationary vane is properly located with respect to the movable one, the moving system will be deflected. Ammeters and voltmeters operating on this principle differ only in the size of wire and the number of turns in the coil carrying the current to be measured.

*Thomson Inclined Coil Instruments.*—A diagrammatic representation of the construction of a Thomson

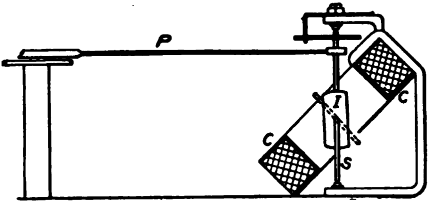


Figure 109.—Principle of Thomson Inclined Coil Instrument.

inclined-coil instrument is shown in Figure 109. The coil *C* carrying the current to be measured is mounted at an angle to the shaft *S* supporting the pointer *P*. A strip, or bundle of strips, of iron *I* is mounted on

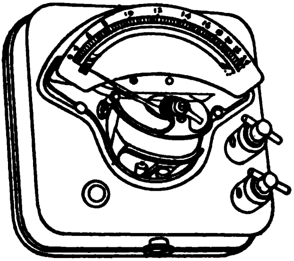


Figure 110.—Thomson Inclined Coil Ammeter, Cover Removed.

the shaft *S* and held by a spiral spring when there is no current in the coil, so that its position is nearly parallel to the plane of the coil. When a current is established in the coil, the vane tends to take up a position with its longest dimension parallel to the magnetic field, which results in the shaft being rotated and the pointer moved over the scale. The moving system will come to rest when the deflecting and restoring forces are equal. The degree of this movement will vary with the ampere-turns in the coil. Ammeters and voltmeters operating on this principle differ only in the size of wire and the number of turns in the coil carrying the current to be measured. A Thomson inclined coil type of instrument with the cover removed is shown in Figure 110.

*Adaptability of Instruments with Soft-Iron Parts.*—The instruments described in the previous three sections may be used in measuring either direct or alternating currents and pressures. Their indications are, however, not reliable when used in direct-current work because they are influenced by outside fields and masses of iron. There is also a lag of the magnetic condition of the piece of soft iron behind the magnetizing force, which results in their indication being lower than the true value as the current in the coil is increasing in value, and higher as the current in the coil is decreasing in value. With alternating current, these objections are not present and the operation of the instruments will be found to be quite satisfactory, provided the outside field, if any, does not change at the same frequency as the current being measured. The inductance of a voltmeter of this type is usually quite high and, as a result, the indication of the instrument will be different for different frequencies, which is a decided disadvantage.

Considerable time, as a rule, is required for the moving system to come to rest after a given current is established in the coil, which is a disadvantage in the operation of the instruments, but can be overcome to a certain extent by attaching a light aluminum vane to the moving system which dampens the movement due to its fan-like action. The scale divisions are not uniform throughout the entire scale.

*Induction Type Ammeters and Voltmeters.*—Instruments of this class depend upon the principle of a rotating magnetic field which is produced by the current to be measured, acting upon a short-circuited secondary, such as a copper or aluminum disk or cup. The Westinghouse ammeters and voltmeters of the induction type have two coils which correspond to the primary and the secondary windings of a current transformer. The primary winding is arranged to be connected in series with the circuit in which the current is to be measured and the secondary winding is short-circuited on itself, as shown in Figure 111.

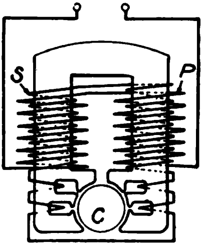


Figure 111.—Diagram of Westinghouse Induction Ammeter.

The resultant action of the two currents in the primary and the secondary windings, which are displaced in phase, produces the effect of a rotating magnetic field. This rotating magnetic field acts upon the copper or aluminum disk, or cup, *C* and causes it to rotate about the axis upon which it is supported against the action of one or more coiled springs. The strength of these springs is so adjusted that a full scale deflection is obtained when a current equal in value to that for which the instrument is designed is passed through the primary winding. This same principle is used in the construction of wattmeters and watt-hour meters and is treated more at length in Chapters VIII and IX. Induction instruments are suitable for alternating-current measurements only, and their indication depends upon the frequency of the current being measured and the temperature, but methods are employed in the better types to compensate for these errors. The scales of such instruments are unusually long and open, frequently covering as much as 300 degrees. A portable induction ammeter with the case removed is shown in Figure 112.

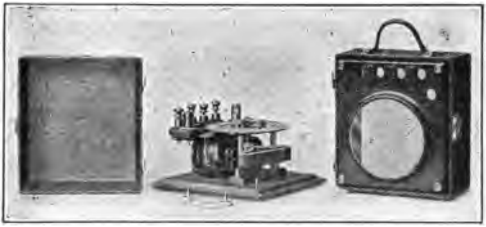


**Figure 112.—Westinghouse Portable Induction Ammeter, Cover Removed.**

### CLASS "C" INSTRUMENTS

*Electrodynamometers.*—The electrodynamometer is, perhaps, the best example of an instrument whose operation depends upon the magnetic force exerted between two coils, both of which have a current in them. The two coils are usually placed at right angles to each other as shown in Figure 113, one of them being fastened rigidly to the frame of the instrument, while the other is supported or suspended and its position controlled by a spring. When a current exists in both coils, the movable one tends to turn into such a position that its magnetic field is parallel to the magnetic field produced by the current in the stationary coil. The movable coil, however, is brought back to its zero position by turning the thumb screw, which is connected to the upper end of the spring, causing the spring to move, and the tortion in the spring exactly balances the tendency for the coil to turn when the thumb screw has been turned through the proper angle. A pointer is fastened to the thumb screw and moves over a graduated scale. The effect produced by a given current through the two coils, which are connected in series, is then read as so many divisions on the scale.

When the instrument is used as an ammeter, the two coils consist of a few turns of large wire, depending, of course, on the current capacity of the instrument. Very frequently the stationary part is composed of two coils having a different number of turns. A full-scale deflection will be produced by a smaller current when the stationary coil having the larger number of turns is used. In some cases the stationary coils are arranged so they can be connected in parallel, series, or used singly, and this changes the current range of the instrument.

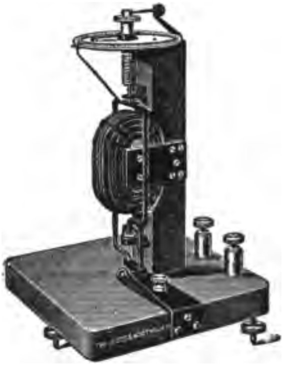


Figure 113.—Electrodynamometer.

When the electrodynamometer is used as a voltmeter, the coils are composed of a large number of turns of small wire. Frequently an additional non-inductive resistance is provided to be used in series with the instrument, which increases its range as a voltmeter.

*Adaptability of Electrodynamometer Instruments.*—The indications of instruments of the electrodynamometer type are influenced by the effect of stray magnetic fields and, as a result, they are not altogether satisfactory for direct-current measurements. This error, however, can be reduced to practically zero if the disturbing effect remains constant—which is not always the case—by taking the average of two readings of the instrument with the current through the coils in the opposite direction in the two cases. The above errors do not occur when the electrodynamometer type of instruments is used in alternating-current measurements, unless the frequency of the disturbing field is the same as the frequency of the current being measured, because the current is continuously reversing in direction.

The inductance of the electrodynamometer voltmeter should be as low as possible in order that a change in frequency of the impressed voltage will cause a small error in the indication of the instrument.

## INSTRUMENTS WHOSE OPERATION DEPENDS UPON THE HEATING EFFECT OF A CURRENT

*Heat Generated in a Conductor Due to a Current in the Conductor.*—When there is a current produced in a conductor, the energy expended in overcoming the resistance of the conductor is manifested in the form of heat. The rate at which the heat is developed in a conductor is proportional to

(a) The resistance of the conductor
(b) The square of the current in the conductor.

Numerous practical applications are made of the heating effect of a current in a conductor, such as electric irons, cooking stoves, heaters, lamps, electric welding, fuses, hot-wire instruments, etc.

*Hot-Wire Instruments.*—The heat generated in a conductor due to a current in it will cause the conductor to expand. The amount of this expansion will depend upon the rise in temperature, which, in turn, depends upon the current in the conductor. The principle upon which one of the hot-wire instruments on the market at the present time operates is shown diagrammatically in Figure 114. A wire *AB* of comparatively high resistance, low-temperature coefficient, and non-oxidizable metal, has one end attached to the plate *C*, then passed over the pulley *P*, which is attached to the shaft *S*, and its free end brought back and mechanically, though not electrically, attached to the plate *C*. The spring *F* keeps the two sections of the wire between the plate *C* and the pulley *P* under tension, it being attached to the plate *C*, which is so guided that it can move only in a direction perpendicular to itself. An arm *G* is also attached to the shaft *S*, it being counterweighted at the upper end by the weight *W*, and bifurcated at the lower end. A fine silk thread *T* has one end attached to one arm of *G*, then passed around a small

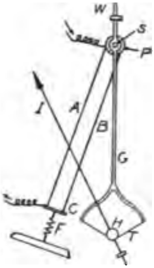


Figure 114.—Principle of Hot-Wire Instrument.

pulley *H,* which is mounted on a shaft that carries a pointer *I,* and finally has its other end attached to the other arm of *G.* The material composing the arms *G* is springy and serves to keep the silk fiber in tension. The current to be measured passes through the section of wire *A,* entering and leaving through two twisted conductors, as shown in the figure. When a current is passed through *A,* it is heated and expands, which results in the tension in the section of wire *A* being less than in the section of wire *B*—they were originally the same—and equilibrium can be restored only by the pulley *P* rotating in a clockwise direction. This rotation of the pulley *P* causes the lower end of the arm *G* to move toward the left, and the silk thread which passes around the pulley *H* causes it to rotate in a clockwise direction, This rotation of the pulley *P* causes the lower end of the arm *G* to move toward the left, and the silk thread which passes around the pulley *H* causes it to rotate in a clockwise direction, and, as a result, the pointer *I* is deflected to the right, it being rigidly connected to the same shaft to which the pulley *H* is connected. In the case of ammeters, except those used in measuring small currents, a low-resistance instrument is used in parallel with a suitable shunt, while in the case of voltmeters a high-resistance instrument is employed.

*Adaptability of Hot-Wire Instruments.*—The operation of the instrument described in the previous section is quite satisfactory, as any change in the temperature of the room in which it is used does not affect the correctness of its indication, since both sections of the wire *A* and *B* are affected to the same extent, which results in no movement of the pointer. In some instruments of this type, no adjustment or compensation is provided for errors due to changes in room temperature. Hot-wire instruments may be used equally well in both alternating- and direct-current measurements, the heating effect of the current being independent of its direction.

## INSTRUMENTS WHOSE OPERATION DEPENDS UPON THE ELECTROSTATIC EFFECT

*Principle of Electrostatic Instruments.*—Electrostatic instruments depend for their operation upon the attraction between electrostatic charges of opposite sign. When a condenser is charged by connecting it to a source of electrical pressure, there is a force tending to draw the plates together. If the construction of the condenser is such that one of the plates may move, the capacity of the combination will increase somewhat as the plates approach each other, thus increasing the amount of the charge on the plates due to a certain pressure and, hence, the attractive force between them. The movable plate may be held in its normal position by a coiled spring, or a gravity control may be used. This principle is used in the construction of electrostatic voltmeters but the forces acting on the moving system are so small for ordinary voltages, say up to 150 volts, that they are not satisfactory. They are, however, well adapted to the measurement of high voltages.

*Electrostatic Voltmeter.*—An electrostatic voltmeter for measuring extremely high voltages is shown diagrammatically in Figure 115. The principal parts of the instrument are the moving elements $M_1$ and $M_2$, the curved plates $B_1$ and $B_2$, the condensers $C_1$ and $C_2$, scale $S$, pointer $P$, and terminals $T_1$ and $T_2$. The plate $B_1$ is connected to the inner plate of $C_1$ and the plate $B_2$ to the inner plate $C_2$. The plates $B_1$ and $B_2$ are curved and so arranged with respect to $M_1$ and $M_2$, which are electrically connected, that a movement of the pointer $P$ over the scale $S$ above zero shortens the gap between the moving elements and fixed plates. The charges on the plates $B_1$ and $B_2$ are of opposite sign and they induce opposite charges on $M_1$ and $M_2$, which results in a force that tends to turn the moving system about its own axis, or into such a position that $M_1$ and $M_2$ are nearer $B_1$ and $B_2$. The movement, however, is restrained by a spiral spring, not shown in the figure and the deflection is indicated by the pointer $P$ on the scale $S$. The form of the two plates $B_1$ and $B_2$ is such that the deflection increases almost directly as the impressed voltage between the terminals $T_1$ and $T_2$. The two condensers $C_1$ and $C_2$ are so constructed that either or both of them may be short-circuited, which results in the pressure required to produce a full-scale deflection being less than when they are in circuit, thus giving the instrument quite a wide range. The condensers and plates shown in Figure 115 are all immersed in oil, which affords a good insulation and also serves to buoy up the moving system, practically removing all the weight from the bearings. Instruments of this type are constructed to measure voltages up to 200,000 volts.

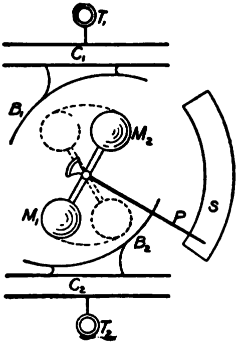


Figure 115.—Principle of Electrostatic Instrument.

*Adaptability of Electrostatic Instruments.*—Electrostatic instruments may be used in both direct- and alternating-current measurements. Their indication, however, is not correct, if calibrated by direct current or low-frequency alternating current, when used in measuring currents of high frequency, such as are used in wireless work. Their operation is not at all satisfactory when the pressure to be measured is low, and for this reason they cannot be used in combination with a shunt in the measurements of current, as the resistance of the shunt would have to be large in order that there be the proper difference in pressure between its terminals to operate the meter, which would mean an excessive loss in comparison to that which would occur if a low-resistance shunt could be used.

# CHAPTER VIII

## CONSTRUCTION AND OPERATION OF WATTMETERS

*Measuring Power by Means of an Ammeter and Voltmeter.*—The power in any part of a direct-current circuit may be determined by measuring the current with an ammeter, and the difference in pressure, by means of a voltmeter, between the terminals of the portion of the circuit in which it is desired to ascertain the power. The power can then be calculated by multiplying the ammeter reading, in amperes, by the voltmeter reading, in volts, or

$$P = E \times I$$

Thus, the power taken by a small motor $M$ may be determined by connecting an ammeter $A$ in series with the circuit to the motor and a voltmeter $V$ across the terminals of the motor, as indicated in Figure 116. The product of the two instrument readings at any instant, in volts and amperes, will give the power, in watts, being supplied to the motor.

When the connections are made as indicated in Figure 116, the ammeter indicates the current through the motor and voltmeter combined. This results in an error in the calculation of the power taken by the motor on a basis of the voltmeter and ammeter readings, and the value of the error will depend upon the relation between the currents in the voltmeter and the motor circuits. When the current taken by the motor, or load, is large in comparison to the current taken by the voltmeter, the current taken by the voltmeter may be neglected, as it produces no appreciable effort upon the indication of the ammeter. For very accurate measurements, however, the current taken by the voltmeter should be subtracted from the value of the current indicated by the ammeter. The voltmeter circuit may be opened and the ammeter indication taken if there is no change in pressure between the voltmeter terminals, which would be the load current alone.

If the voltmeter be connected across the line before

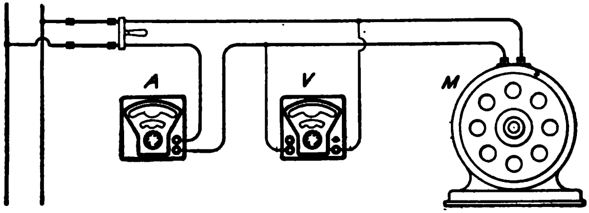


Figure 116.—Measuring Power Taken by a Motor.

the ammeter—that is, the ammeter would be between the voltmeter connections and the motor—the indication of the voltmeter would not be the true value of the pressure between the terminals of the motor or load, as there would be a certain drop in pressure across the ammeter. The drop across the ammeter should be subtracted from the voltmeter indication in order to obtain the true pressure across the load. With a low-resistance ammeter, this drop is very small and may be neglected except when very accurate results are desired.

In the case of an alternating-current circuit, the product of the current and the pressure as measured above will not give the value of the true power unless the current and the pressure are in phase. Corrections cannot be made for the current taken by the voltmeter and for the drop over the ammeter, as in the direct-current circuit, because the current through the voltmeter is not exactly in phase with the load current, nor is the drop in pressure between the terminals of the ammeter in phase with the pressure across the load. With instruments having low inductance, the error due to a difference in phase of the current and the pressure is very small and may be neglected except in very accurate measurements.

*Wattmeters.*—A wattmeter is an instrument for measuring power and its indication depends upon the combined effects of the load current, or a definite part of it, in the circuit in which the power is being measured, and the pressure acting upon the load. The majority of wattmeters in use at the present time may be divided into two classes, depending upon the fundamental principle upon which they operate. These classes are (1) electrodynamometer wattmeter; and (2) induction wattmeters.

*Principle of the Electrodynamometer Wattmeter.*—The principle of the electrodynamometer wattmeter can be illustrated by reference to Figure 117, which shows an electrodynamometer with one coil *S* connected in series with one of the leads to the load and the other coil *P* connected across the line. The coil connected in series with the load is wound as though it were to be used as an ammeter, and usually consists of a few turns of large wire, while the coil connected across the line is wound as though it were to be used as a voltmeter and consists of many turns of fine wire. These two coils are usually called the *series* and the *pressure* coils, respectively.

The operation of the electrodynamometer shown in Figure 117 may be explained as follows: The current taken by the lamps, or load, passes through the series coil of the instrument and produces a certain magnetic effect, which changes in value and direction with the current. The current in the pressure coil, or circuit, will vary with the pressure between the terminals of the circuit, since the resistance of the circuit remains constant and neglecting any inductance the circuit may have and, as a result, there is a magnetic field produced about the pressure coil which will change in strength and direction with the pressure acting on the circuit. The deflection of the moving coil of an electrodynamometer is proportional to the product of the magnetic effects of the current in the two coils, and since these magnetic effects are proportional to the load current and voltage, respectively, the indication of the instrument, when it is properly balanced, must be proportional to the product of *E* and *I*, or it may indicate watts when the instrument is calibrated and the scale properly marked.

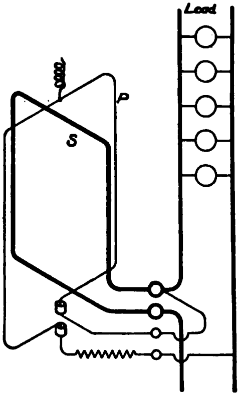


Figure 117.—Electrodynamometer Used as a Wattmeter.

When an instrument of this type is connected in a circuit in which there is a fluctuating current, due to a varying load, and the pressure between the terminals of the pressure circuit remains constant, the indications will vary directly with the load current. The magnetic effect of the current in the pressure circuit remains constant, since the pressure acting on the circuit remains constant and there is no change in the resistance, which results in a constant current. If the load current remains constant and there is a change in the value of the pressure between the terminals of the pressure circuit, the magnetic effect of the current in the series coil remains constant and the magnetic field due to the current in the pressure coil changes with the pressure, which results in the indication of the instrument varying with the voltage. If the load current and pressure current both change in value the indication of the instrument, when a balance is obtained, will be proportional to the product of the two currents.

In an alternating-current circuit, the current and the pressure are continuously changing in value and periodically reversing in direction. If the current and the pressure pass through zero value at the same time, the direction of the deflecting force acting on the moving system will remain constant; but if the current and the pressure pass through zero value at different times, the deflection force acting on the moving system will be in one direction for a portion of each alternation and in the reverse direction for the remainder of the alternation. In the first case the indication of the instrument is proportional to the average force acting on the moving system during each alternation, which is proportional to the true power supplied to the load. In the second case the indication of the instrument is proportional to the average force acting on the moving system during each alternation, which is equal to the difference in the force acting in one direction and that acting in the opposite direction, or it is proportional to the true power supplied to the load. If the current in the series coil and that in the pressure coil be displaced in phase by 90 degrees, the force acting on the moving system in one direction will be exactly equal to the force acting on it in the opposite direction for one alternation, or the resultant force will be zero and the moving system will not be deflected from its position of rest. The period of the moving system of the instrument must be somewhat greater than the period of the current passing through its windings in order that the moving system may come to rest at a definite deflection. It is not possible to have the load current and pressure over the load displaced in phase by 90 degrees so the resultant force acting on the moving system will never be equal to zero, but it may be very small.

*Roller-Smith Wattmeter.*—The principle of the Roller-Smith wattmeter is the same as that of the

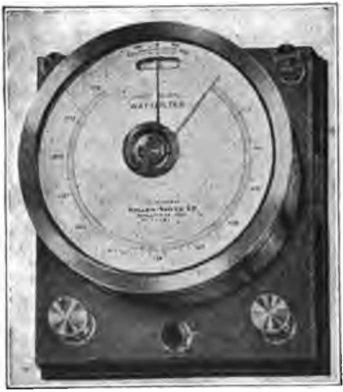


Figure 118.—Roller-Smith Wattmeter.

dynamometer wattmeter just described, except that the construction is modified to make the instrument portable and more suitable for commercial work. In this wattmeter the heavy current winding is composed of two coils, which are supported in suitable frames and enclose a coil composed of fine wire. This coil is mounted on a shaft with pointed ends resting in jeweled bearings. Two volute springs serve to hold the coil in its zero position and balance the turning effort of the coil when there is a current in its winding and the current coils. These two springs also serve to conduct the electricity into and from the movable coil, instead of the mercury cups, as in the previous case. A balance is obtained by turning the torsion head in the center of the top of the instrument until the needle attached to the movable coil is indicating no deflection; the indication of the pointer attached to the torsion head is then noted and, if the instrument has been calibrated and the scale properly marked, should correspond to the power in watts. The general appearance of the instrument is shown in Figure 118.

*Weston Wattmeter.*—The principle of the Weston instrument is practically the same as the instrument described in the previous section. No torsion head, however, is used and the pressure coil, instead of being maintained at practically the same position for

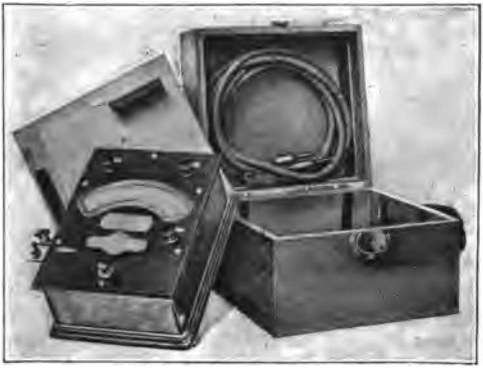


Figure 119.—Weston Wattmeter.

all indications of the instrument, may rotate about its axis through an angle of approximately 90 degrees. Two volute springs serve to conduct the electricity into and from the movable coil. They also hold the movable coil in its zero position and offer the opposing force to that produced by the magnetic effects of the currents in the pressure and the series coils. This form of instrument has an advantage over the two previous forms described in that no manipulation is necessary to bring about a balance and the needle attached to the moving system moves directly to a definite position on the scale, which should be marked to correspond to the power the instrument is measuring. A Weston wattmeter is shown in Figure 119 and diagrammatically in Figure 120. In Figure 120, $C_1$ and $C_2$ represent the two current coils which are connected in series between the two posts marked *1* and *2*. The pressure connections may be made between the terminals *3* and *5*, or between *3* and *6*. The resistance between the terminals *3* and *6* is usually just half the value of the resistance between the terminals *3* and *5*. A full-scale deflection may then be produced by either of two voltages with the same current in the series coil.

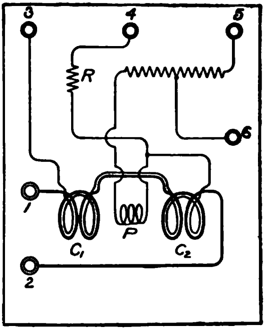


Figure 120.—Diagram of Weston Wattmeter.

A top view of the Thomson indicating wattmeter manufactured by the General Electric Company is shown in Figure 121. The current coil in this instrument is inclined.

*Compensated Wattmeter.*—The power indicated by a wattmeter will be in error just the same as the value of the power determined by the voltmeter and ammeter readings is in error, due either to the current taken by the voltmeter or the drop across the ammeter, unless some means be provided which will counteract or compensate for this error. Compensation for the current taken by the pressure circuit in the Weston wattmeter is provided in the following manner: The pressure circuit is always to be connected across the line after the current coil, which results in the current coil carrying the current that passes through the pressure coil. This current in itself would produce a deflection of the moving system, even though there be no load current through the current coil, and, as a result, the meter would always indicate a higher value of power than that actually taken by the load. If, however, the pressure circuit be wound around the series coils the same number of times there are turns in the series coils and this winding be so connected in the pressure cir-

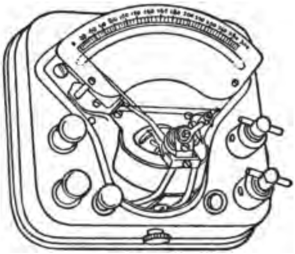


Figure 121.—Thomson Inclined Coil Wattmeter.

cuit that the magnetic effect of the pressure current in it will be opposite to that of the current in the series coils, the magnetic field produced by the pressure current passing through the series coils will be completely annulled and there will be no deflection of the moving system until there is a load current through the series coil. The turns of wire wound around the series coils are called *compensating turns,* and they are shown in Figure 120. When a separate source of current and pressure are used in calibrating a wattmeter, there should be no compensating turns in the pressure circuit and terminal *4* should be used instead of terminal *3.* The resistance of the pressure circuit should be the same in both cases, and a non-inductive resistance *R,* whose resistance is equal to that of the compensating turns, is connected in the pressure circuit and replaces the compensating turns. An interior view of the Westinghouse dynamometer type wattmeter is given in Figure 122.

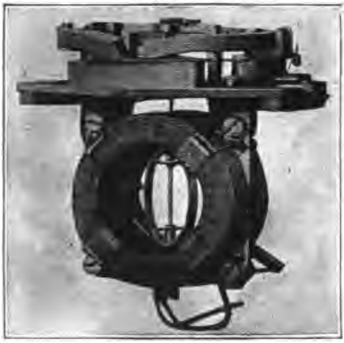


**Figure 122.—Westinghouse Dynamometer Wattmeter.**

*Adaptability of Electrodynamometer Wattmeters.* —Electrodynamometer wattmeters will operate on either direct- or alternating-current circuits. Their indication on direct-current circuits will, however, be influenced by stray magnetic fields; and, in the majority of cases, a reversal of current through the coils, although the value of the current remains constant, will result in a difference in indication. The effects of stray magnetic fields are eliminated in what is called the *astatic* type of instrument. This consists of a double combination of series and pressure coils so interconnected that the effect of any stray field on one combination is just the reverse of what it is on the other combination with zero resultant effect. When the instruments are used in alternating-current circuits, these errors are very much reduced—provided the disturbing field is not alternating and of the same frequency as the current in the coils of the instrument—since the current passes through the coils in one direction for the same length of time it passes through them in the opposite direction. It is necessary that the inductance of the pressure circuit be very small in order that the current through it be as near in phase with the pressure between its terminals as possible. The error due to the inductance of the pressure circuit increases as the value of the power factor decreases.

*Principle of the Induction Type Wattmeter.*—The induction type wattmeter depends upon the action of a rotating magnetic field upon a metal disk or cup which is rotated, due to the action of the field, against the torque of a coiled spring. The operation of this type of meter in general may be explained by reference to Figure 123. The essential parts are a metal disk, or cup, *C;* a pressure circuit *P;* a series coil *S;* and a pointer, attached to the pivoted shaft upon which the piece of metal *C* is mounted, which moves over a suitable scale. Neither the pointer nor the scale is shown in the figure. The deflecting force acting on the piece of metal is counteracted by the action of a coil spring, not shown in the figure. The pressure coil is very highly inductive, so the current in the coil lags approximately 90 degrees, or one-quarter of a period, behind the pressure coil. There is a magnetic field produced by the current in the pressure coil which is in phase with the current, and the variations in this magnetic flux through the metal disk, or cup, induce eddy currents which are displaced in phase from the flux by one-quarter of a period.

The series coil *S* is non-inductive, or approximately so. The current in the series coil produces a magnetic flux through the piece of metal *C* which is in phase with the current. The eddy current, produced by the current in the pressure circuit, and the flux due to the series current, are in phase when the pressure acting on the pressure circuit and the current in the series coil are in phase. With this phase relation between the flux and the eddy currents, the reaction between them will be a maximum, and the piece of metal will be rotated due to the reaction.

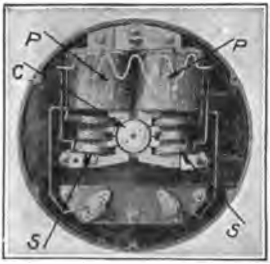


Figure 123.—Westinghouse Induction Type Meter.

The eddy currents in the piece of metal C are proportional to the current in the pressure circuit, which is proportional to the pressure between the terminals of the pressure circuit. Similarly, the flux due to the load, or series current, is proportional to the value of the current, and, as a result, the reaction between the two gives a deflecting force which is proportional to the product of the line pressure and the load current, which is the power.

When the current and the pressure are not in phase, the eddy currents and flux do not reach their maximum value at the same time nor do they pass through zero value at the same time, which results in a decrease in the effective torque acting on the moving element, and the indication in this case is proportional to $EI\cos\theta$, where $\theta$ is the phase displacement of the current and the pressure with respect to each other. Hence, the induction wattmeter, when properly constructed and adjusted, will give an indication which is proportional to the true power in circuits whose power factor is different than unity.

*Westinghouse Induction Wattmeter.*—The operation of the Westinghouse induction wattmeter may be explained by reference to the diagrammatic sketch of its magnetic circuit and windings which is shown in Figure 124. The magnetic circuit of this instrument is very similar to the magnetic circuit of the induction ammeter manufactured by the same company and shown diagrammatically in Figure 111. The windings in this case are different than in the

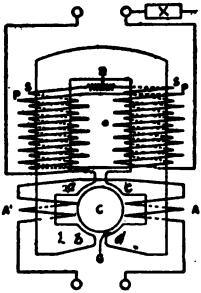


**Figure 124.—Diagram of Westinghouse Induction Wattmeter.**

case of the ammeter, there being two windings connected to the line instead of one. The two coils marked *P* are connected in series and form the *pressure circuit* of the meter, while the coils marked *A* are in series with the load and are called the *current coils.* The pressure circuit has a high inductance and the current in this circuit is displaced in phase from the pressure by almost 90 degrees. The path of the magnetic flux produced by the pressure current is indicated by the dotted lines in Figure 125, and the path of the magnetic flux produced by the current in the series coils by the dotted lines in Figure 126. The pressure current makes the two projections *a-b* of opposite polarity to the two projections *c-d,* and the series current makes the polarity of *a-c* different than the polarity of *b-d.* Since the two currents are displaced in phase, the resultant magnetic field rotates, and there is a torque produced on the moving element, which causes a deflection, but the movement of the system is opposed by spiral springs.

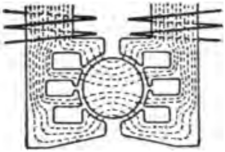


Figure 125.—Magnetic Flux Produced by Pressure Current.

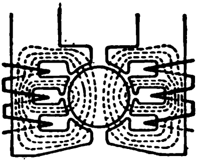


Figure 126.—Magnetic Flux Produced by Series Current.

In describing the operation of the instrument, the assumption has been made that the current in the pressure circuit and the pressure producing it are displaced in phase 90 degrees, or one-quarter of a period. This result cannot be obtained in practice with the arrangement of the coils thus far described, for the pressure coil has some resistance and there must be a power component of the current in the pressure circuit to take care of the hysteresis and eddy current losses in the iron through which the magnetic flux produced by the pressure current passes. The current in the series coil and resulting flux are not exactly in phase. As a result of these relations it is not correct to assume that the flux due to the pressure current shall differ from that due to the series current by exactly one-quarter period. The error resulting from such an assumption is small when the power factor of the load is high, but it increases as the value of the power factor decreases.

One of the simplest methods of adjusting the operation of the meter so it will indicate correctly on both inductive and non-inductive loads consists in winding the core of the voltage coil with a short-circuited coil whose resistance is adjustable. The short-circuited coil acts as the secondary of a transformer, of which the regular voltage coil is the primary. This method is used in the Westinghouse wattmeter shown diagrammatically in Figure 124, in which *S-S* represents the coils short-circuited by the resistance *R*.

*Adaptability of Induction Wattmeter.*—The induction wattmeter may be used only on alternating-current circuits. Their construction is, as a rule, more rugged than other types of instruments; they are capable of developing a high deflecting torque; they are not subject to the influence of stray magnetic fields to any great extent, unless the fields have the same frequency as the current in the meter; they are not affected to any great extent due to changes in temperature; they have longer scales than other instruments; and slight errors due to changes in frequency between certain limits.

*Polyphase Wattmeters.*—Polyphase wattmeters consist of two or more wattmeter elements mounted on the same shaft, and arranged so that the current and the pressure circuit of each element may be connected to their respective circuits.

# CHAPTER IX

## CONSTRUCTION AND OPERATION OF WATT-HOUR METERS

*Classification of Watt-Hour Meters.*—The watt-hour meter is an instrument which sums up all of the successive instantaneous values of the energy supplied by a source of energy through the meter to a distributing circuit or load and registers the sum of these values by means of a suitable recording device. All the standard types of watt-hour meters in use in this country are a combination of a small motor driving a small generator, or a motor-generator set. The construction of the motors is such that the torque developed by them is proportional to the product of the current and the pressure, or the power, while the torque required to drive the generator is proportional to its speed. With this relation, it is obvious, that the speed of the combination is proportional to the power. The number of revolutions in a given time is proportional to the average power during that time, or the revolutions per unit time, say one minute, vary as the average power during that time. If the value of the power which must pass through the meter continuously for one minute, or the watt-minutes in order to produce one revolution of the rotating element is known, the meter may be used in measuring the energy supplied to a load in watt-minutes by noting the total number of revolutions of the disk and multiplying this value by the watt-minutes required per revolution, which will give the total energy.

Commercial watt-hour meters may be divided into three groups according to the principle upon which the motor element of the meter operates. These groups are: (1) *Commutator Type;* (2) *Mercury Motor Type;* and (3) *Induction Type.*

*Principle of the Commutator Type Watt-Hour Meters.*—The commutator type watt-hour meter operates on the same principle as the electrodynamometer indicating wattmeter with the modification that instead of the deflection of the moving element being opposed by coiled springs, a continuous rotation is obtained by connecting different coils successively in circuit by means of a small commutator and set of brushes. The moving element of such a meter is somewhat similar in construction to the armature of a direct-current motor. The load current is passed through coils and produces a magnetic field in which the armature rotates. The strength of the field varies with the load current and, hence, the torque, acting on the armature, varies directly as the load current. The current through the armature winding, or pressure circuit, varies with the pressure between the terminals of the circuit and, as a result, the torque, acting on the armatures, varies with the pressure. Since both of these effects are present at all times when the meter is operating, it is obvious the torque acting on the moving element, or armature, is proportional to the power.

In order that the speed of the meter may vary with the torque acting on the armature, or the power, it is necessary that there be some load connected to the shaft upon which the armature is mounted, whose counter torque varies with the speed. Such a counter torque is provided by mounting on the armature shaft a disk, or cup-shaped piece of metal, which is arranged to revolve between the poles of several permanent magnets. These magnets and the disk are shown in the lower part of Figure 127. The production of this counter torque, in brief, is as follows: There is a constant magnetic flux produced by each of the permanent magnets and, as the disk revolves in this constant flux, there are eddy currents induced in it which are proportional to the speed of the disk. The counter torque is proportional to the product of the eddy currents and the magnetic flux passing through the disk; and, since the eddy currents are proportional to the speed, the counter torque must be proportional to the speed. The counter torque increases and decreases with the speed or it changes as the torque acting on the armature changes. When there is an increase in current in either the armature or the series windings, there is an increase in torque on the armature, and the speed increases until the counter torque just balances the torque on the armature. With a decrease in either of the currents or both, there is a decrease in speed until the two torques again balance each other. It is thus seen, neglecting the friction of the moving parts, that the speed of the armature is proportional to the power and the meter should operate correctly at all loads.

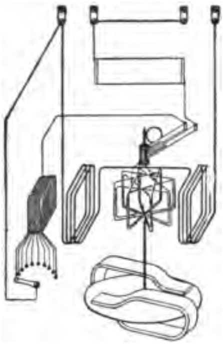


Figure 127.—Diagram of Commutator Type Watt-Hour Meter.

The torque, acting on the armature of the meter times the time it acts, determines the number of revolutions of the armature in the given time, or

$$\text{revolutions vary as torque} \times \text{time}$$

but the torque is proportional to the power being consumed by the load, or it is equal to the product of the power and some constant, or

$$\text{torque varies as } E \times I$$

Substituting this expression for torque in the first equation gives

$$\text{revolutions vary as } E \times I \times \text{time}$$

or

$$\text{revolutions} \times \text{a constant} = E \times I \times \text{time}$$

The product on the right-hand side of the above equation represents electrical energy. Representing the constant by $K$ and the revolutions by $N$, we may write the equation as follows:

$$\text{electrical energy} = KN$$

The energy transmitted to the load will produce a certain number of revolutions of the armature, and if the value of $K$ in the last equation is known, the total energy can be computed by multiplying the revolutions by the value of $K$.

The number of revolutions of the armature is transmitted through a suitable system of gearing to the dials mounted on the front of the meter. The pointers on the different dials are interconnected by gears in such a way that one revolution of the pointer on one dial will result in the pointer on the next higher dial moving over one division, or one-tenth of a revolution. The speed of the armature is so adjusted in the design of the meter and by the permanent magnets that one revolution corresponds to a definite amount of energy. It is customary to connect the armature to the dial so the meter reads direct in one of the energy units such as the watt-hour or kilowatt-hour.

*Compensation for Friction in Commutator Type Watt-Hour Meters.*—A certain amount of torque is required to overcome the friction of the moving parts and, as a result, all of the torque developed in the armature, as described in the previous section, is not available for producing rotation. The friction in a well-designed meter is very small, yet if some means is not provided for overcoming it, the meter will not register on very light loads.

This counter torque, due to friction, is overcome by what is called a *compensating torque,* which is developed as follows: A coil is connected in series with the armature, or pressure circuit, and mounted in such a position that its plane is parallel to the current, or series coil. The direction of the current around this coil is the same as the load current in the series coils, or the magnetic effects of the compensating and the series coils are in the same direction. This results in there being a torque produced on the armature, even though there be no current through the series coils; and if the magnetic effect of the compensating coil is properly adjusted, the armature will start to revolve just as soon as there is any current in the series coils. The magnetic effect of the compensating coil on the armature is usually adjusted in either of two ways: first, its position with respect to the armature may be changed, which will increase or decrease the torque, depending whether the coil is moved toward or away from the armature; or, second, the number of active turns in the compensating coil may be changed, its position with respect to the armature remaining constant.

Examples of the first method are found in the General Electric and Westinghouse watt-hour meters and of the second method in the Duncan and Columbia watt-hour meters. A diagram of the Duncan watt-hour meter is given in Figure 127, which shows clearly the connection of the compensating coil and the method of varying its magnetic effect by changing the number of active turns.

*Creeping of Commutator Type Watt-Hour Meters.*—There is a current through the pressure circuit of a watt-hour meter all the time when the meter is in service. If the adjustment of the compensating coil is such that there is more torque acting on the armature, no current in the series coils, than is required to overcome the counter torque due to friction, the armature of the meter will revolve, or *creep,* as it is called. In some cases the adjustment of the compensating coil may have been made under ideal conditions as to vibration; but when the meter is installed, the vibration reduces the friction and, as a result, the meter creeps.

Another cause of creep may be due to an excessive voltage acting on the pressure circuit, which increases the current in the armature and compensating coil with the result that the torque, due to the pressure current, is more than enough to overcome the friction and the moving element will rotate, even though there is no current in the series coils. The tendency for the meter to creep is frequently reduced by means of a small metal clip of iron attached to the edge of the disk, and when this clip comes under the damping magnets, the motion of the disk is retarded.

*Three-Wire Direct-Current Watt-Hour Commutator Type Meter.*—In measuring the energy delivered by a three-wire direct-current circuit, one three-wire, or two two-wire watt-hour meters may be used. The principle of operation of the three-wire direct-current meter is no different than the two-wire meter, the only difference is in their construction. There are usually two series coils, one in each outside lead, and the pressure circuit is connected between one outside lead and the neutral, or between the outside leads.

The turning torque acting on the moving element is due to the combined action of the currents in the two coils and it is proportional to the combined power in the two sides of the three-wire system, assuming the voltage between the neutral and each of the outside wires is the same.

The drop in pressure in the neutral wire of a three-wire system tends to increase the drop over the lighter-loaded side, and if the neutral current is greater than the current on the lighter-loaded side, there will be a greater voltage over the lighter load than at the generating end of the line. If the pressure coil of the watt-hour meter happens to be connected on the lighter-loaded side, the indication of the wattmeter will be too high; while if the pressure circuit is connected to the other side, the indication is too low. In some cases the pressure circuit is connected between the outside leads, but this does not result in the meter indicating correctly because the currents on the two sides are not delivered at the same voltage only when the loads are balanced.

*Commutator Type Watt-Hour Meters on Alternating-Current Circuits.*—In the discussion of the electrodynamometer wattmeter it was pointed out that the deflecting torque was proportional to the power when the pressure circuit was non-inductive. Since the principles of operation of the commutator type watt-hour meter as used on direct-current circuits are practically the same as those of the electrodynamometer type wattmeter, the rotating torque is not strictly proportional to the power delivered to the load. A watt-hour meter with an inductive pressure circuit will give too high an indication on circuits in which the current lags the pressure and too low an indication in a circuit in which the current leads the pressure.

The above errors in the indication of the meter are adjusted by changing the series circuit in such a way that the phase relation of the series current and voltage shall be exactly equal to the phase relation between the armature current and the voltage on non-inductive load. This phase relation of the series current is brought about by shunting a part of the total load current around the series coils by means of a non-inductive resistance. Since there is some inductance in the series winding, the current through it will lag the pressure over the divided circuit, which is in phase with the current and is numerically equal to the product of the current through the non-inductive resistance and the value of the resistance in ohms.

*Principles of the Mercury-Motor Watt-Hour Meter.* —The principle upon which the motor element of the mercury-motor watt-hour meter operates was discovered by Barlow almost one hundred years ago and may be explained by reference to the diagram shown in Figure 128. A disk *D* is mounted on a horizontal axis so that it may rotate freely between the poles of a permanent magnet *M.* Two conducting springs *S, S* are mounted in such a way that they bear lightly on the outer edge of the disk and the shaft upon which it is mounted, respectively. If a current be sent through the disk in the direction indicated by the arrow *I,* there will be a torque developed due to the reaction between the current and the magnetic flux

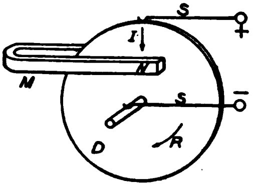


Figure 128.—Principle of Mercury-Motor Meter.

from the permanent magnet, which will cause the disk to rotate in the direction of the arrow *R*. The direction in which the disk will rotate for a given direction of current and flux may be determined by the use of the left-hand, or motor, rule as described in Chapter II. The torque acting on the disk is proportional to the value of the current, the field strength remaining constant; or if both the field strength and the current change, the torque is proportional to the product of the two.

The application of this principle in the Sangamo direct-current watt-hour meter may be readily understood by reference to Figure 129, which shows diagrammatically the theoretical relation of the various parts. The current of electricity carried by the circuit in which energy is to be measured enters the contact $C_1$, passes through the comparatively high resistance mercury *H* to the disk *D*, across the disk to the mercury *H*, and out at contact $C_2$. The magnetic field is produced by an electromagnet whose winding is connected across the circuit in which the energy is to be measured, which results in the strength of the field varying with the voltage. The torque developed by the reaction of the current in the disk upon the magnetic field is proportional to the power being supplied to the load. By connecting such a motor to an eddy-current generator, or damper, which requires a driv-

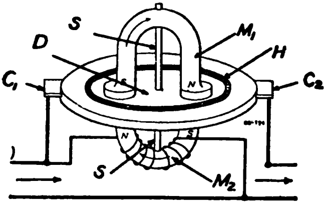


Figure 129.—Diagram of Sangamo Mercury-Motor Meter.

ing torque directly proportional to the speed of rotation, the combination may be used as a meter.

In the construction of these meters, a small wooden float is attached to the disk immersed in the mercury chamber, and the buoyancy of the mercury which the disk and float replace is a little greater than the weight of the complete moving element. This results in a slight upward pressure on the upper end of the armature shaft instead of on the lower end, as in the majority of meters. The magnetic circuit of the electromagnet, which is placed directly beneath the mercury chamber, is improved by means of a circular ring of laminated sheet steel wound edgewise under heavy pressure and imbedded in the upper part of the moulded mercury chamber. The construction of the interior of the mercury chamber is very similar to that of the invertible ink-well, and as a result it is almost impossible to spill the mercury, no matter in what position the meter may be placed. The disk carrying the current through the mercury chamber is slotted so as to guide the current at right angles across the magnetic field of the electromagnet, thus rendering all of the current effective in producing torque.

*Compensation for Friction.*—Compensation for friction or light-load adjustment is accomplished by means of a thermocouple, shown at *A* in Figure 130. This thermocouple consists of two pieces of dissimilar metals having slotted ends and fastened under the screws *B* and *C*. A number of turns of fine wire are wound around the couple and connected in series with the pressure circuit of the meter. The thermocouple is heated, due to the pressure current in the thermocouple winding, which results in a small thermoelectromotive force being produced in the couple. This thermoelectromotive force is used in producing a current through the mercury chamber in the same direction as the load current and, as a result, there will be a torque acting on the moving element even though there be no load current through the meter. The value of the thermocouple current through the meter element is adjusted by means of a resistance connected in parallel with the meter element. This resistance consists of the copper wire *F* and the resistance wire *G*, which are joined by the sliding connection *E*, and their left-hand ends connected to the terminals of the couple. The degree of compensation is adjusted by moving the connection *E* along the wires *F* and *G*. If it is moved to the left, there is more current from the thermocouple shunted through it and less goes through the motor element of the meter; while if it is moved to the right, less current is shunted and more passes through the motor element of the meter. With the first adjustment the torque is decreased, and with the second adjustment it is increased.

When the meter is connected in the negative lead, the action of the thermocouple will be different than when it is connected in the positive lead, unless the connections of the thermocouple be reversed. The connections shown in Figure 130 are correct when the meter is connected in the positive side of the circuit; but if the meter is installed in the negative side

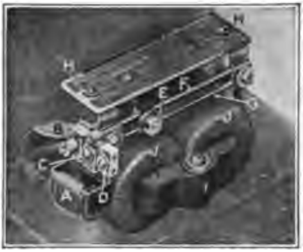


Figure 130.—Light-Load Adjustment on Sangamo Mercury-Motor Watt-Hour Meter.

of the circuit, the thermocouple should be connected to terminals *C* and *D* instead of to *B* and *C*.

*Creeping of Mercury-Motor Watt-Hour Meters.*—Mercury-motor meters are less subject to the effects of vibration than the commutator type and are not so likely to creep due to this cause. An increase or decrease in pressure will, however, cause them to creep as it causes an increase or decrease in the effect of the thermocouple current. The tendency of the meter to creep may be reduced by attaching a small piece of magnetic material to the damping disk.

*Sangamo Three-Wire Mercury-Motor Meter.*—The Sangamo three-wire meter consists of two motor elements mounted on the same armature shaft, one element being placed directly above the other. The circuits of a sangamo three-wire meter are shown in Figure 131.

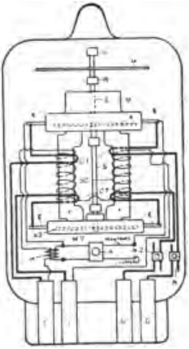


**Figure 131.—Diagram of the Sangamo Three-Wire Mercury-Motor Meter.**

*Sangamo Mercury-Motor Ampere-Hour Meter.*—The ampere-hour meter is usually used in measuring the quantity input or output of a storage battery in ampere-hours. The principle upon which the sangamo mercury-motor ampere-hour meter operates is the same as the watt-hour meter made by the same company, except a strong permanent magnet is employed in producing the magnetic flux instead of the electromagnet. With a permanent magnet the speed is proportional to the current passing through the meter and the dial may be made to read directly in ampere-hours by properly calibrating and adjusting the meter. The front view of a Sangamo ampere-hour meter with the cover removed is shown in Figure 132. Different dials and gears can be used with this meter so that one revolution of the pointer will correspond to any desired number of ampere-hours. The meters are arranged so the pointer turns clockwise when the battery is being charged and counter-clockwise on discharge. By means of a special device, called a *variable resistor*, the speed of the meter in one direction may be made different than in the opposite direction. The object of this arrangement is to take care of the losses in the battery, and the difference in speed in the two directions may be adjusted by merely turning a screw in the bottom of the mercury chamber. The variable resistor consists of a rectangular piece of conducting metal mounted in the mercury chamber, with its longest dimension approximately parallel to the path of the load current through the chamber, and so arranged that it may turn through a small

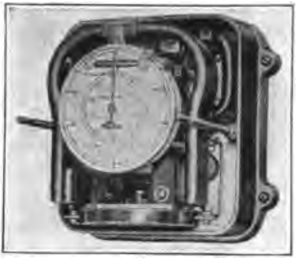


Figure 132.—Sangamo Ampere-Hour Meter, Cover Removed.

angle about the armature shaft. As the disk in the mercury chamber rotates, it tends to carry the mercury and piece of metal around with it, but the movement of the piece of metal is limited by stops mounted in the mercury chamber. When this piece of metal stops in one of its extreme positions, it conducts a greater part of the total current across the mercury chamber than it does when in the other extreme position, which results in the disk carrying a larger part of the total current when rotating in one direction than in the other, hence the meter runs faster for a given total current in one direction than for the same current in the reverse direction.

*Mercury-Motor Watt-Hour Meter for Alternating-Current Circuits.*—Since the pressure circuit of the direct-current mercury-motor meter has such a high inductance, it is impossible to use the meter on alternating-current circuits. The principle of the mercury-motor meter for alternating-current measurements is as follows: The load and voltage circuits are interchanged, and the magnetic flux is produced by the load current, while the current through the disk is taken from the secondary of a potential transformer whose primary is connected across the line. Since the magnetization of the electromagnet is due to the load current, the resulting magnetic flux will be in phase with the load current and vary in value with it. The secondary circuit of the transformer, including the circuit through the motor element, is practically non-inductive and, as a result, the secondary current is in phase with the secondary voltage. The secondary current will rise and fall in value with the primary voltage, as the secondary voltage is displaced in phase from the primary voltage by 180 degrees. When the load is non-inductive, the load current and the pressure are in phase and the magnetic flux through the disk will rise and fall with the current the disk is carrying. Thus, on a non-inductive load there will be a torque acting on the motor element which is proportional to the product of the current and pressure, or *EI.*

If, however, there is a phase displacement of the load current and pressure, the magnetic flux through the disk and the current the disk is carrying do not pass through zero and maximum values at the same time, and the effective torque acting on the motor element is decreased in the ratio of the power factor.

The compensation for friction or light-load adjustment in this type of meter is taken care of as follows: A second secondary winding of few turns is placed around the core of the transformer and connected to an auxiliary winding placed about the electromagnet. This auxiliary winding is composed of two parts which are connected in parallel to the secondary winding through a special rheostat so arranged that the relation of the current in the windings may be varied. These two windings are so connected that they produce opposite magnetizing effects and the resultant effect will depend upon which effect predominates or the adjustment of the rheostat.

*Principle of Induction Type Watt-Hour Meters.*—The fundamental principles upon which the induction type watt-hour meter operates is practically the same as that of the induction motor, or, in general, it depends upon a torque being produced in a movable closed-circuited secondary by means of a rotating, or a shifting, magnetic field. The magnetic field in which the rotating element is placed is a combination of the magnetic fields produced by the load current and the current in the pressure circuit. As a result of the action of these fields, eddy currents are induced in the secondary which react upon the fields and produce the torque tending to turn the movable element. The torque developed is a maximum when the current in the pressure circuit and that in the series coils are displaced in phase by 90 degrees or—assuming the pressure current and the pressure producing it are displaced in phase by 90 degrees—when the pressure over the load and the load current are in phase. The value of this torque is proportional to the product of $E$ times $I$; and when the line pressure and load current are displaced in phase, the torque is proportional to $EI\cos\theta$.

The pressure winding of such meters are wound so as to have as high an inductance as possible, which results in the current being displaced in phase from the pressure by almost 90 degrees. The series windings are composed of a comparatively small number of turns and have a very small inductance. These two windings are placed upon suitable magnetic circuits so arranged that the magnetic effect of the two combine and produce a rotating, or shifting, field in which the movable element is placed. The secondary element consists of a disk, or cup-shaped piece, of aluminum, which is mounted on a shaft so it may freely rotate in the field and at the same time between the poles of several permanent magnets. These permanent magnets together with the disk provide a load which requires a driving torque directly proportional to the speed; and, since the developed torque is proportional to the true power, the speed of the meter will vary directly as the power.

It is impossible to cause the current in the pressure circuit to lag the pressure by exactly 90 degrees on account of the resistance of the winding and the iron losses in the iron composing the magnetic circuit about which the winding is placed; and, as a result, the magnetic flux of the pressure circuit and the load current are not displaced in phase with respect to each other by exactly 90 degrees. It is imperative that this displacement be 90 degrees in order that the meter will indicate correctly on both inductive and non-inductive loads, which is accomplished by introducing a low-resistance short-circuited winding in the field of the pressure circuit, or by changing the reluctance of the pressure circuit. When this winding is properly adjusted, the meter indication should be proportional to the power regardless of the power factor.

Induction meters are not free from friction and some compensation for friction must be provided so that the meter will read correctly on low loads. This is accomplished by the use of closed-circuited loops, or secondaries, which are arranged with respect to the magnetic field of the pressure circuit in such a way that they produce an unbalanced or unsymmetrical condition of the field, thus producing a slight torque on the moving element. This effect may be varied—generally by changing the position of the secondaries—and the compensation for friction properly adjusted.

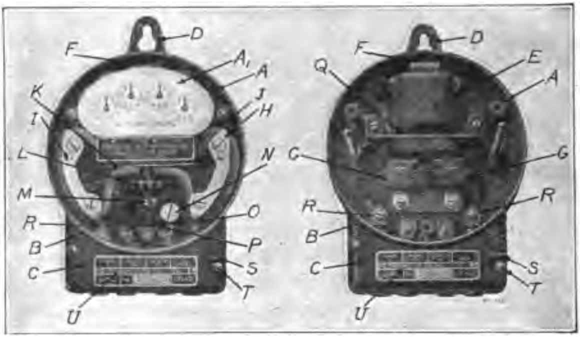


Figure 133. Figure 134.

**Figure 133.—Front View Sangamo Type H Watt-Hour Meter, Cover Removed.**

**Figure 134.—Front View Sangamo Type H Watt-Hour Meter, Cover, Damping Magnets and Register Removed.**

There are a number of different types of induction watt-hour meters on the market at the present time and, although the fundamental principle upon which they operate is the same, they differ somewhat in their construction.

*Sangamo "Type H" Watt-Hour Meter.*—The moving element of this meter consists of an aluminum disk, mounted on a hard brass shaft, with two slots cut at diametrically opposite points. If there is any tendency for the meter to creep, the disk will rotate until one of these slots enters the field of the pressure or the shunt winding, where it will lock. This locking action is not sufficient, however, to prevent the meter starting and recording correctly on light loads. A front view of the meter with the cover removed is shown in Figure 133; and a second front view with the disk, damping magnets, and recording train, or register, removed is shown in Figure 134, from which it will be seen the disk is acted upon at the back by the pressure and the series coils, producing rotation, and in front by two permanent magnets which retard its motion.

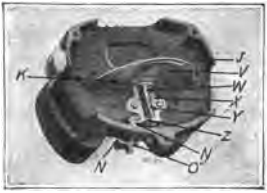


**Figure 135.—Light-Load Adjustment Sangamo Type H Watt-Hour Meter.**

The letters in Figures 133, 134, and 135 refer to the following parts:

*A*—base of the meter; $A_1$—dial; *B*—terminal box; *C*—terminal box cover; *D*—hanging lug; *E*—pressure coil; *F*—pressure magnet; *G*—series coils; *H*—magnet clamp; *I*—case stud; *J*—main grid; *K*—disk; *L*—full-load adjustment disk; *M*—full-load adjustment clamp screw; *N*—micrometer light-load adjustment screw; *O*—light-load adjustment clamp screw; *P*—lower bearing screw; *Q*—lagging coil or winding; *R*—

**binding posts; *S*—sealing lug; *T*—sealing screw; *U*—type and number plate; *V*—copper vane; *W*—shaft on which vane is mounted; *X*—supporting bracket for vane; *Y*—spring; *Z*—sector for imparting motion to shaft from screw *N*.**

The full-load adjustment is accomplished by a soft-iron disk mounted on an adjustable screw so it may be moved nearer to or farther away from the space between the poles of the permanent magnets. When the iron disk is raised, more of the magnetic flux is shunted through it and less passes through the dampened disk *K*, thus decreasing the induced eddy currents and hence the load on the meter.

The flux through the magnetic circuit is made to lag the current in the series coils by means of what is called a *lagging coil,* which consists of a few turns of wire about the pressure magnetic circuit connected to a low adjustable resistance. By adjusting this resistance, the shunt field is brought into the proper phase relation with respect to the series current.

The compensation for friction, or light-load, adjustment is accomplished by moving a copper vane *V*, Figure 135, which is located in the gap of the pressure magnetic circuit. A movement of this vane either way from its central position will cause a slight unbalancing of the magnetic field due to the pressure current, which produces a torque tending to rotate the moving element. This vane is carried on a shaft marked *W*, Figure 135, which may be rotated against the action of a coiled spring *Y* by means of the sector *Z* and the micrometer light-load adjusting screw *N*. The inner end of the screw *N* rests against the sector and a slight rotary movement of the screw will turn the shaft upon which the vane is mounted through a very small angle.

*Fort Wayne Electric Works "Type K" Single-Phase Watt-Hour Meter.*—This meter is manufactured at the Fort Wayne Works of the General Electric Company and is the same as the General Electric

"Type I-14." A front and a rear view of the meter are shown in Figures 136 and 137, in which the letters refer to the following parts:

*A*—cast shoe supporting magnets; *B*—full-load micrometer adjusting screw; *C*—magnet shoe clamping screws; *D*—test terminal; *E*—cover sealing screw for terminal chamber; *F*—light-load micrometer adjusting screw; *G*—clamping screw for starting plate; *H*—starting plate; *I*—current coils; *J*—pressure coil; *K*—terminals; *L*—holes for mounting screws.

The moving element of this meter consists of a light aluminum disk mounted on a short shaft constructed from a special alloy which combines strength, lightness, freedom from corrosion, and resistance to wear. The lower end of this shaft has inserted in it a hardened steel pivot which is ground and highly polished. This pivot rests in a cup-shaped jewel bearing, which is also very highly polished, thus reducing friction and wear to a minimum. The rotating and the retarding torques act on opposite sides of the disk.

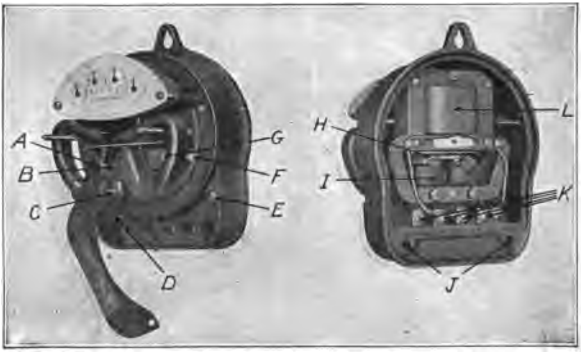


Figure 136. Figure 137.

Figure 136.—Front View Fort Wayne Type K Watt-Hour Meter, Cover Removed.
Figure 137.—Back View Fort Wayne Type K Watt-Hour Meter.

The full-load adjustment is accomplished by moving the permanent magnets toward or away from the center of the disk. The movement of these magnets is controlled by the full-load micrometer adjusting screw *B*. This screw passes through the shoe supporting the magnets and into the meter base. The part of this screw entering the meter base is threaded left hand and the part in the magnet shoe is threaded right hand. If this screw be turned counter-clockwise, the magnets are moved toward the center of the disk and the speed increases; and if it be turned clockwise, the magnets are moved toward the edge of the disk and the speed decreases.

The compensation for friction, or light-load, adjustment is accomplished by moving a rectangular metal punching, represented by *H* in Figure 137, which is placed over the cores of the series winding between the windings and the disk. The position of this punching may be moved by the light-load micrometer adjusting screw *F* and the desired compensation obtained. There is a second punching of copper attached to the punching *H*, or starting plate as it is called, which may be moved with respect to the punching *H*. Inductive load adjustment, or the lagging of the meter, is obtained by moving the copper punching with respect to the starting plate *H*. Two holes are cut in the disk which prevent creeping due to fluctuations in voltage or vibration.

*Polyphase Induction Watt-Hour Meters.*—Polyphase induction watt-hour meters consist of two or more motor elements mounted on a common shaft and driving one or more eddy-current generators. The electrical and the magnetic circuits of the different elements should be independent of each other within the meter. The operation of the motor elements should be such that their combined torque for a given power will be the same on both balanced and unbalanced loads. A polyphase induction watt-hour meter with the cover removed is shown in Figure 138.

*Torque-Weight Ratio.*—In speaking of the torque of a meter as generally applied, one means the turning moment of the force acting on the moving element tending to make it turn. The value of this torque is usually expressed in terms of millimeter-grams, and a torque of 5 millimeter grams is equal to a force of one gram acting upon a lever arm 5 millimeters long. It is always desirable that a watt-hour meter have a reasonably high torque in order that its accuracy on light loads may not be influenced to any great extent by an increase in the friction of the bearings or of the commutator and the brushes. It is not so much the torque which counts in the end as it is the ratio of the torque to the weight of the moving element on the bearings. For example, one wattmeter might have a higher torque than another, but if the weight of its moving element on the bearings was greater than the first, its ratio of torque to bearing weight might be less, and the effective torque available for overcoming any friction in the meter would be less as the friction

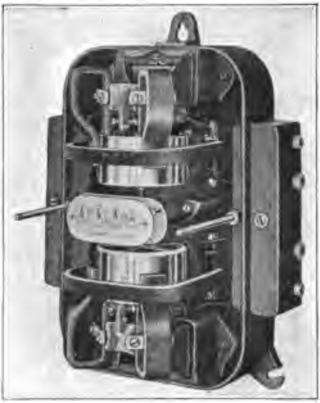


Figure 138.—Polyphase Induction Watt-Hour Meter.

bears a definite relation to the bearing weight of the moving element.

The weight of the moving element in most induction watt-hour meters is much less than in direct-current meters, and as a result there is less wear on the jewel bearings and less driving torque is necessary.

*Meter Humming.*—A peculiar sound is often heard in alternating-current meters, which is called *humming* and it is very frequently mistaken by the inexperienced for the actual operation of the meter, when no energy is being used. This humming sound is caused by a vibration of the moving element of the meter in synchronism with the current in the windings, or it may be due to a vibration of loose laminations or operating parts, but should not be mistaken as an indication that the moving element is revolving.

*Meter Capacities.*—Direct-current watt-hour meters are usually constructed to carry the total current in the line except in a few cases when shunts are used. The resistance of the current circuit of the majority of direct-current meters is such that it would require too high a resistance shunt in the line in order to produce the necessary drop to operate the meter. The Columbia Meter Company and the Sangamo Electric Company manufacture meters which may be used with shunts very successfully. In the case of alternating-current meters they are usually self-contained for capacity up to about 150 amperes and 650 volts; while for higher currents and voltages, transformers are used in combination with 5-ampere 115-volt meters. The use of these transformers permits the safe handling of meters in service as the attendant is not exposed to contact with high voltages.

*Prepayment Watt-Hour Meter.*—The operation of the prepayment watt-hour meter proper is the same as that of the ordinary watt-hour meters. It is in reality a direct- or alternating-current watt-hour meter to which has been attached a special device which will automatically close the electrical circuit, when a coin is inserted and the knob turned, and keeps the circuit closed until the energy which the coin deposited will purchase has been used, when the circuit is automatically opened. The interior of a prepayment meter manufactured by the Fort Wayne Electric Works is shown in Figure 139, and the circuit closing switch is clearly shown at the top of the figure. This form of meter affords many advantages and is readily adapted to many different conditions. The most common applications of the meter are found where the customer's credit has been found bad; where the customer is chronically complaining about his bill; where the customer is not financially able to make the required deposit; where the customer is employing electric signs for advertising purposes or

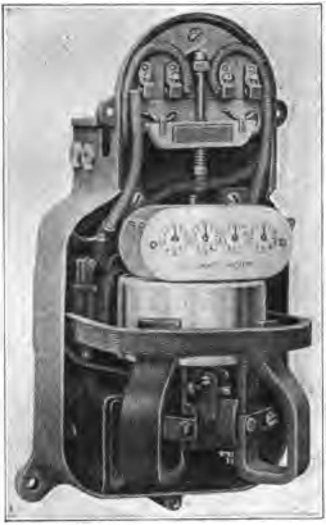


Figure 139.—Prepayment Meter, Cover Removed.

operating an automobile charging station. In addition to this, many customers who do not come under the above classifications prefer to pay for things "a little at a time" and make no exception to their electric light bill. For all of these conditions, the prepayment meter affords the ideal medium whereby the bad credit customer may be again supplied with current without fear that the bill will not be paid. The fault finder pays for the current so gradually that he no longer imagines the amount as excessive and the complaints are reduced. Customers employing electric signs for advertising may wish to spend a certain amount in such form of advertising. This can be done by depositing at any time such amount as the customer desires to put into the advertising. After the lights are started, the meter will automatically switch off the lights when energy to the amount deposited has been used. This will enable the customer to keep an accurate account of the expense of advertising.

Other advantages that appeal to those selling current are the elimination of bad accounts and the saving in postage, collectors' fees, bookkeeping, etc. These and many other uses of the meter will readily be appreciated.

# CHAPTER X

## METHODS OF DISTRIBUTING ENERGY, AND THE MEASUREMENT OF POWER AND ENERGY

*General Classification of Systems.*—Electrical energy is distributed by means of either direct or alternating current. Alternating current may be distributed single-phase, two-phase, or three-phase, and when more than a single phase is used, it is called *polyphase distribution.* When the distribution is at a practically constant potential and variable current, it is called a *constant-potential system;* and when the current remains practically constant and the voltage acting on the circuit changes with the load, it is called a *constant-current system.* The constant-potential systems are in more general use than the constant-current systems. The best example of the constant-current system is found in the series arc lighting circuits, which may be either direct or alternating.

Direct current and single-phase alternating current are distributed in either two-wire or three-wire systems. The two-wire system may be a constant-current or a constant-potential system. A three-wire system has an additional wire to that used in the constant potential two-wire system, and the potential of this third wire is intermediate between the two outside wires. The advantages of the three-wire system lie in its saving in copper in the distribution of a given amount of energy, and its flexibility in that current may be obtained from such a system at two voltages and in some cases—when the voltage between the neutral and outside wires is not the same—three voltages. The actual copper required in a three-wire 220- and

110-volt system to transmit a given power with the same loss is only three-eighths as much as is required in a two-wire 110-volt system. In the three-wire system as used for power and lighting, it is the aim of the company supplying energy to keep the voltage between the neutral and each outside wire the same, and such a system is said to be *balanced* when these voltages are the same and each outside wire is carrying the same current. When the system is balanced, there is no current in the neutral wire and the only loss is that in the two outside wires.

Polyphase alternating-current systems have their various phases interconnected in such a way as to usually require only three or four wires. The methods usually employed in measuring the power and energy by wattmeters and watt-hour meters, respectively, are given in the following sections.

*Power and Energy in Two-Wire Direct- and Alternating-Current Circuits.*—The power in any two-wire direct-current circuit is equal to the product of the current in the circuit and the pressure between the terminals of the circuit, or

$$P = EI$$

in which $P$ is the power in watts; $E$ is the pressure between the terminals of the circuit in volts; and $I$ is the current in the circuit in amperes.

The power in any two-wire alternating-current circuit, in which the current and the pressure are in phase, is equal to the product of the effective pressure between the terminals of the circuit in volts and the effective current in the circuit in amperes. The same formula is used in calculating the power as for a direct-current circuit.

The power in any two-wire alternating-current circuit, in which the current and the pressure are not in phase, is not equal to the product of the effective value of the pressure between the terminals of the circuit and the effective current in the circuit; but this product must be multiplied by a factor called the *power factor,* which is equal to the cosine of the angle $\theta$ between the current and the pressure, in order to get the true power, or

$$P = EI\cos\theta$$

in which $P$ is the true power in watts; $E$ is the effective pressure in volts; $I$ is the effective current in amperes; and $\cos\theta$ is the power factor. In an alternating-current circuit, the product of the effective pressure and the effective current is called the *apparent power,* which will be equal to the true power when the value of the $\cos\theta$ is unity or the pressure and current are in phase.

The power in a two-wire direct- or alternating-current system may be measured by a wattmeter having

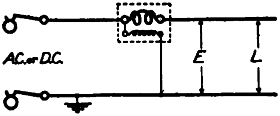


Figure 140.—Watt-Hour Meter Having Single Current Coil Connected in Ungrounded Side of a Two-Wire Circuit.

its current coil connected in series with one side of the circuit and its pressure coil connected across the circuit. The pressure circuit should be connected so the current through it passes through the current coil.

The connections of a watt-hour meter with a single-current coil is shown in Figure 140. If either side of the circuit is grounded, the current coil should be connected in the ungrounded side. When the current coil is connected in the grounded side of the meter, a second ground may occur on the load side of the meter without interfering with the service, and, as a result, only part of the current would flow through the current coil of the meter and the meter would record only part of the energy expended in the circuit. Both sides of the line may be connected through separate current coils in the meter, as shown in Figure 141, when neither side of the circuit is grounded.

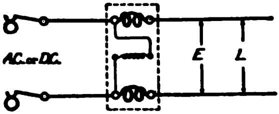


Figure 141.—Watt-Hour Meter Having Two Separate Current Coil Connected in Opposite Sides of a Two-Wire Circuit.

*Power and Energy in Three-Wire Direct-Current and Single-Phase Alternating-Current Circuits.*—Any three-wire direct-current or single-phase alternating-current circuit may be thought of as being composed of two two-wire circuits, and the total power and energy may be measured by means of two wattmeters or watt-hour meters, one being connected in each circuit. The current coil of one of the meters should be connected in one of the outside wires and the current coil of the other meter in the remaining outside wire.

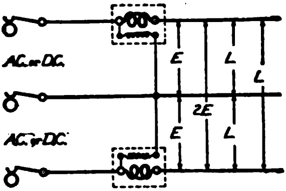


Figure 142.—Two Watt-Hour Meters Connected in the Outside Leads of a Three-Wire Circuit.

The pressure coils of the two meters should be connected from the outside wire, in which their current coils are connected, to the neutral wire, as shown in Figure 142. When wattmeters are used, the pressure coil should be connected on the load side of the current coil. The total power, or energy, will be equal to the sum of the wattmeter or watt-hour meter readings, as the case may be.

Since the pressure between the neutral and each of the outside wires is practically the same in the ordinary three-wire system, a single watt-hour meter with two current coils, which act upon a single pressure coil, may be used in measuring the energy delivered to the loads. The pressure circuit may be connected between the neutral and one outside wire or it may be connected between the two outside wires. The first method is not quite so accurate as the second method, but it is the one generally used in direct-current meters; while the second method is employed in alternating-current meters, as the losses in their pressure circuits are much smaller than in direct-current meters. The connections of a three-wire watt-hour meter are given in Figure 143, in which the pressure circuit is connected between the neutral and one outside wire.

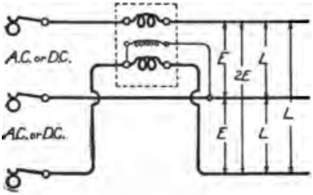


**Figure 143.—Three-Wire Watt-Hour Meter Connected in a Three-Wire Circuit.**

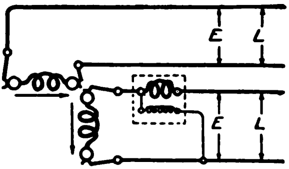


**Figure 144.—Watt-Hour Meter Connected in One Phase of a Two-Phase Four-Wire System.**

*Power and Energy in a Two-Phase Four-Wire System.*—A two-phase four-wire system is in reality nothing more than two single-phase two-wire systems, which may or may not be entirely independent. The power or energy may be measured by means of two wattmeters, or watt-hour meters, one connected in each of the two phases as in the case of a single-phase system.

A two-phase four-wire system in which the two phases are entirely independent of each other is shown in Figure 144, and a single-phase watt-hour meter is shown connected in one of the phases.

An interconnected two-phase four-wire system is shown in Figure 145, in which the center points of the two sources of electromotive force are connected together. In this case the electromotive force between the junction of the two phases and any one of the outside terminals, assuming the electromotive force in each phase is the same, will be equal to one-half the electromotive force per phase. The pressure between any two adjacent terminals will be proportional to the diagonal of a right-angle triangle, each side of which is proportional to one-half the electromotive force per phase, or

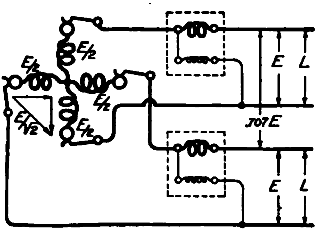


Figure 145.—Two Watt-Hour Meters Connected in a Two-Phase Four-Wire Interconnected System.

$$E' = \sqrt{\frac{E^2}{2} + \frac{E^2}{2}}$$

$$= \frac{E}{\sqrt{2}}$$

$$= .707\,E$$

in which $E'$ represents the electromotive force between adjacent terminals.

The power or energy in such a system, when the loads are balanced, may be determined by multiplying the reading of a single wattmeter or watt-hour meter, connected in either of the phases, by two. When the loads are unbalanced, two wattmeters or watt-hour meters will be required.

*Power and Energy in Two-Phase Three-Wire System.*—A two-phase source of electrical energy may have one terminal of each of the two phases connected to a common conductor which serves for both phases and only three wires will be required, as shown in Figure 146. The voltage between the outside wires in such a system will be proportional to the diagonal of a right-angle triangle whose sides are proportional to the electromotive force of the two phases. When the electromotive force of each of the phases is the same, the voltage between the outside wires $E'$ is given by the following equation:

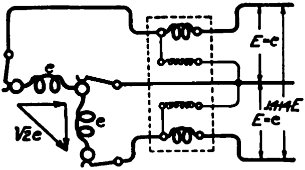


Figure 146.—Two-Phase Watt-Hour Meter Connected in a Two-Phase Three-Wire System.

$$
\begin{aligned}
E' &= \sqrt{E^2 + E^2} \\
&= \sqrt{2}E \\
&= 1.414\,E
\end{aligned}
$$

The current in the common wire of a two-phase three-wire system is equal to the vector sum of the currents in the two outside wires, which are displaced in phase by 90 degrees. When the two outside currents are equal, the value of the current $I_n$ in the common wire is given by the following equation:

$$\begin{aligned} I_n &= \sqrt{I^2+I^2} \\ &= \sqrt{2}\,I \\ &= 1.414\,I \end{aligned}$$

in which $I_n$ is the neutral current and $I$ the outside currents. The power or energy in such a system, when the loads are balanced, may be determined by multiplying the reading of a single wattmeter or watt-hour meter, connected in either of the phases, by two. On an unbalanced load two wattmeters or watt-hour meters will be required and the sum of their readings will be an indication of the total power or energy as the case may be. The connections of a single watt-hour meter for measuring the energy in a two-phase three-wire system are given in Figure 146. A watt-hour meter for this purpose really consists of two single-phase watt-hour meters constructed on a common shaft and operating a single registering device.

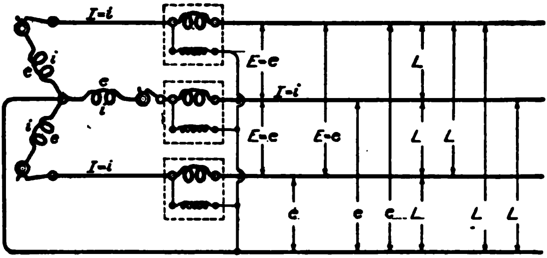


Figure 147.—Three Single-Phase Watt-Hour Meters Connected in a Three-Phase Four-Wire System.

*Three-Phase Connections.*—In a three-phase system of distribution, there are three electrical pressures differing in phase by 120 electrical degrees, and the same phase relation exists between the three currents when the system is perfectly balanced. A three-phase system may be operated as three independent phases by using six wires, two to each phase; but it is customary to interconnect the different phases and use only three or four wires—depending upon the system—which results in quite a saving in the amount of copper required to transmit a given total power with a certain allowable loss. The three phases may be interconnected in either of two ways which are called the *star* and *delta* connections.

In the star connection, one terminal of each of the three phases is connected to a common point and the three remaining terminals are connected to the line wires, as shown at the left in Figure 149, forming what is called a three-wire three-phase star connection. In some cases a fourth wire is run from the junction of the three phases, as shown at the left in Figure 147, forming what is called a four-wire three-phase star connection.

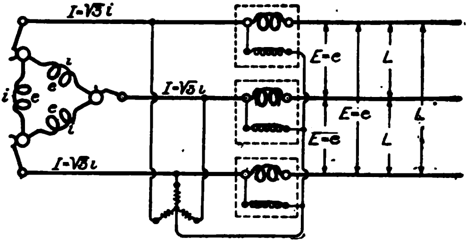


**Figure 148.—Three Single-Phase Watt-Hour Meters Connected in a Three-Phase Three-Wire System.**

In the delta connection, the three different phases are interconnected, as shown in Figures 148 and 150, and the three junctions connected to the three lines. This is known as the three-wire delta connection.

*Power and Energy in a Balanced Three-Phase Four-Wire System.*—The power or energy in a three-phase four-wire system, when the loads are perfectly balanced, may be determined by multiplying the reading of a wattmeter or single-phase watt-hour meter, whose current coil is connected in one of the main wires and whose pressure coil is connected from that wire to the fourth or common wire, by three. When the load is unbalanced, three meters will be required, as indicated in Figure 147.

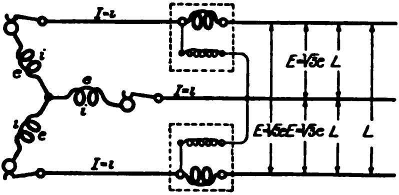


**Figure 149.—Two Single-Phase Watt-Hour Meters Connected in a Three-Phase Three-Wire System.**

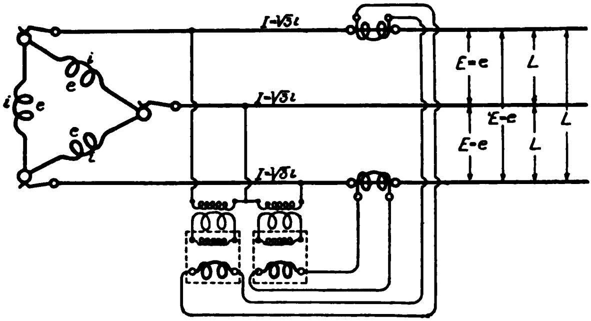


**Figure 150.—Two Single-Phase Watt-Hour Meters Connected in a Three-Wire Three-Phase System with Instrument Transformers.**

*Power in Balanced Three-Phase Three-Wire Systems.*—From an inspection of Figure 147, it is evident that the electrical pressure between any two of the phase wires is not equal to the electrical pressure generated in the different phases at the source of supply, but it is equal to the vector difference of the electrical pressure generated in the two phase windings across which the connections are made. When such a system is electrically balanced, the electrical pressure between any two of the phase wires is equal to the $\sqrt{3}$ times the electrical pressure generated in each of the phase windings. Representing the phase pressure by $E$ and the pressure in the phase winding by $e$, we may write the following equations:

$$E=\sqrt{3}\,e$$

or

$$e=\frac{E}{\sqrt{3}}$$

The current in the different phase wires is numerically equal to the current in the phase windings to which the different wires are connected, for a star connection. Representing the current in the phase wire by $I$ and the current in the corresponding phase winding by $i$, we may write the following equation:

$$I=i$$

The true power in any one of the phase windings is given by the following equation:

$$p=ei\cos\theta$$

in which $p$ is the true power in watts; $e$ is the electrical pressure in each phase winding; $i$ is the current in each phase winding; and $\cos\theta$ is the power factor of the particular phase being considered. Substituting the values of $e$ and $i$ as given in the first equations, we have

$$P=\frac{E}{\sqrt{3}}I\cos\theta$$

In a balanced three-phase star-connected system, the total power is equal to three times the power in any one of the phase windings, or

$$
\begin{aligned}
P &= 3\frac{E}{\sqrt{3}} I\cos\theta \\
&= \sqrt{3} EI\cos\theta \\
&= 1.73 EI\cos\theta
\end{aligned}
$$

in which $E$ is the electrical pressure between the phase wires; $I$ is the current in the phase wires; $\cos\theta$ is the power factor of any phase, as they are all the same if the system is balanced as assumed.

From an inspection of Figure 148, it is evident that the current in any phase wire is not equal to the current in any one of the phase windings at the source of supply, but it is equal to the vector difference of the currents in the two phase windings to which the phase wire is connected. When the system is balanced, the current in any one of the phase wires is equal to the $\sqrt{3}$ times the current in the phase windings, or

$$I = \sqrt{3}\, i$$

or

$$i = \frac{I}{\sqrt{3}}$$

The electrical pressure between any two phase wires is numerically equal to the pressure in the phase winding to whose terminals the phase wires are directly connected, or

$$E = e$$

The true power in any one of the phase windings is given by the following expression:

$$p = ei\cos\theta$$

in which the letters represent the same quantities as in the previous case, or star connection. Substituting the same quantities as given above for the star connections, we have

$$P = E\frac{1}{\sqrt{3}}\cos\theta$$

In a balanced three-phase delta connected system, the total power is equal to three times the power in any one of the phase windings, or

$$P = 3 \times \frac{E}{\sqrt{3}} I\cos\theta$$
$$= \sqrt{3} EI\cos\theta$$
$$= 1.73 EI\cos\theta$$

which is exactly the same equation as used in calculating the total power in a balanced three-phase star-connected system. It must be understood that these equations for the total power hold only for balanced loads.

*Measuring Power and Energy in an Unbalanced Three-Phase Four-Wire System.*—The total power or energy in a balanced three-phase four-wire system may be determined by measuring the power or energy in any one of the three phases by means of a single wattmeter or watt-hour meter, and then multiplying this reading by three. When the system is unbalanced, this method will not work and it is then necessary to measure the power or energy in each of the three phases by means of three wattmeters or watt-hour meters, with the current coil of one meter connected in each of the phase wires and their pressure coils between the phase wire in which the current coils are connected and the neutral, as shown in Figure 147, the total power or energy being equal to the combined indication of the three meters.

*Measuring Power and Energy in a Three-Phase Three-Wire System.*—The power or energy in a three-phase three-wire delta- or star-connected system may be measured by three wattmeters or watt-hour meters, by establishing an artificial neutral point, as shown in Figure 148. This neutral point is established by means of what is called a **Y** box, which consists of three equal non-inductive resistances with one terminal of each of them connected together, which will form the neutral point when the remaining ends of the resistances are connected to the three-phase wires. The impedance of the pressure coils of the meters connected to the neutral point should not be too large in comparison to the value of the resistance of the three coils forming the **Y** box, as the pressure between the neutral point and the different phase wires is likely to be disturbed if such is the case.

The **Y** box may be dispensed with when three meters are available having pressure circuits of the same impedance, by merely joining the three ends of the pressure circuits which would be normally connected to the neutral of the **Y** box. This method of establishing a neutral is subject to error when the higher harmonics are present and the voltage between the neutral point and the phase wires may be increased to a value greater than .707 $E$. If the meters used are not affected by the change in wave form due to the higher harmonics, the three meters will indicate the total true power on balanced or unbalanced loads even if the pressure is increased, as there is a proportionate decrease in the value of the power factor.

The power or energy in a three-phase three-wire star- or delta-connected system may be measured by means of two wattmeters or watt-hour meters, the current coil of one of the meters being connected in one of the phase wires, and the current coil of the other meter in one of the remaining phase wires, and the pressure circuits of each meter being connected between the phase wire in which its current coil is connected and the remaining or third wire, as shown in Figure 149. When such a connection is properly made, the algebraic sum of the two meter readings will give the total power or energy, as the case may be, independent of whether the load be balanced or not, or of the power factor. In dealing with such a system, the middle wire might be thought of as a common return for the two outside wires, as shown in Figure 149, in which case the current returning to the source is equal to the algebraic sum of the currents in the two outside wires flowing from the source toward the load. Now, if the middle wire be thought of as composed of two conductors in parallel, one returning the current flowing out in the upper wire and the other returning the current flowing out in the lower wire, the system is equivalent to a two-phase system and two wattmeters will read the total power.

If the current in the system should lag the pressure by more than 60 degrees or the power factor becomes less than .5, the torque acting on one of the meter elements will be in the opposite direction to what it was when the power factor was greater than .5, and the total power or energy is then equal to the difference in the readings of the two meters.

Three-phase watt-hour meters are usually made with two separate motor elements mounted on the same shaft, and the combination automatically records the total amount of energy. When the power factor becomes less than .5, the torque developed in the motor element of one meter tends to produce rotation in the opposite direction to that produced in the other motor element; and the effective torque is equal to the difference of the two, or it is proportional to the total true power.

When two wattmeters are connected in a three-phase system for measuring the total power, one of them will read backward if the power factor becomes less than .5, and the current or pressure coil connections must be reversed in order that the needle be deflected in the proper direction over the scale. If the connections of one meter be reversed, in order that it read correctly as just explained, the total true power is equal to the difference in the two meter readings.

Very frequently the power factor of a system is not known and, as a result, there may be some doubt as to whether the sum or the difference of the two wattmeter readings should be taken as the total true power, in which case the following simple test may serve in determining whether the sum or difference of the readings should be used. First, connect the two meters so their moving elements are deflected in the proper direction; then change the connection of the pressure circuit of each wattmeter in turn from the third phase wire to the phase wire in which the series coil of the other wattmeter is connected and note the direction in which the moving-system of each meter is deflected. If the moving-system of each of the meters is deflected in the same direction after the change in the pressure connections as before, the sum of the meter readings should be taken as the total true power; while, if the deflection of one meter is reversed when the connections are changed, the difference in the two meter readings should be taken as the total true power.

The proper method of connecting the pressure circuits of a three-phase watt-hour meter are explained in Chapter XII.

*Transformers for Wattmeters and Watt-Hour Meters.*—It is customary to use transformers in combination with wattmeters and watt-hour meters when the value of the current exceeds the capacity of the ordinary meters or when the voltage becomes excessive. The use of these transformers increases the flexibility of the system and should always be used on high-voltage circuits even, if the current is not excessive, for the purpose of insulation. Care should always be exercised in connecting the transformers to make sure that the secondary current delivered to the meters is in the same direction as if the meter windings were connected directly to the live wires. The connections of two current transformers and two potential transformers in combination with two wattmeters or watt-hour meters for measuring the power in a three-phase system are given in Figure 150.

If three meters are used, three currents and three potential transformers will be required. The primaries of the potential transformers must be connected star in order that the pressure acting on each be the same as the pressure that would act on the pressure circuit of the wattmeters if connected directly to the circuit. If the secondaries are connected star, the neutral of the primary star connection must be connected to the neutral of the source, either by a conductor or by grounding, in order to maintain a balance. When the secondaries are connected delta, it is not necessary to connect the neutral of the primary star to the neutral of the source, because the closed delta formed by the secondaries maintains the balance.

*Power Factor in a Three-Phase System.*—The power factor of a balanced three-phase system is defined as the power factor of each of the phases. The value of this power factor for a balanced system can be determined by the following equation:

$$\text{tangent}\,\theta = \sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}$$

in which $P_1$ and $P_2$ represent the readings of two wattmeters connected so as to measure the total power. $P_1$ represents the larger reading and $P_2$ represents the smaller reading which may be either positive or negative as previously described. These values substituted in the above equation gives the value of the tangent $\theta$ and the angle $\theta$ can then be determined by reference to a table of trigonometric functions and also the cosine, which is the power factor.

# CHAPTER XI

## CALIBRATION OF GALVANOMETERS, AMMETERS, VOLTMETERS, AND WATTMETERS

*Primary Standards.*—In the accurate measurement of resistance, current, and pressure, use is made of what are called *primary standards of resistance and electromotive force.* These primary standards should always be carefully calibrated by some reliable testing laboratory at the time of purchase, and then re-checked from time to time in order to detect any change in their accuracy, if such a change should occur.

The recognized authority for electrical standards in this country is the Bureau of Standards of the Department of Commerce and Labor at Washington, D. C., where a set of standards is carefully preserved. Other laboratories located in various parts of the country maintain standards which they keep in agreement with those maintained at the Bureau of Standards.

The standards of resistance and electromotive force are used in combination with a potentiometer in checking other resistances and electromotive forces and measuring current. A number of resistances of different values and different current capacities should be provided so that currents and resistances covering a wide range may be measured.

The standard of electromotive force usually selected is the Weston cell, on account of its negligible temperature coefficient, its high degree of accuracy, and its permanence in calibration. It is customary to have two standard cells: one to be used as a regular working standard, and the other to be carefully preserved and used only in checking the working standard.

*Secondary Standards.*—While the results obtained by the use of the potentiometer are considered the most accurate for direct-current calibration, yet the process is rather slow and for this reason it is customary to provide instruments having a greater precision than the ordinary types which may be used in checking the working instruments. These instruments are called *secondary standards* and they should be checked at intervals, depending on the amount they are used, by means of the potentiometer. Two standard direct-current instruments are shown in Figures 151 and 152. These secondary standards are usually provided with special scales and several multipliers and shunts, so that a full-scale deflection may be produced by different pressure in the case of the voltmeters and different currents in the case of ammeters.

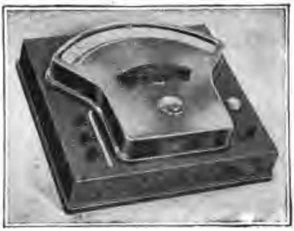


Figure 151.—Standard Direct-Current Voltmeter.

The secondary standards for alternating-current measurements are generally of those types which operate on the electrodynamometer or Kelvin balance principles. When such secondary standards are used, they may be checked by means of the primary standards for direct current and then used in checking and calibrating the portable alternating-current instruments.

Since the power in an alternating-current circuit is equal to the product of the current and the pressure only when they are in phase, it is impossible to calibrate a wattmeter when connected in an alternating-current circuit by means of an ammeter and a voltmeter unless the power factor of the circuit is unity. For this reason it is customary to provide a standard wattmeter of the electrodynamometer or balance type

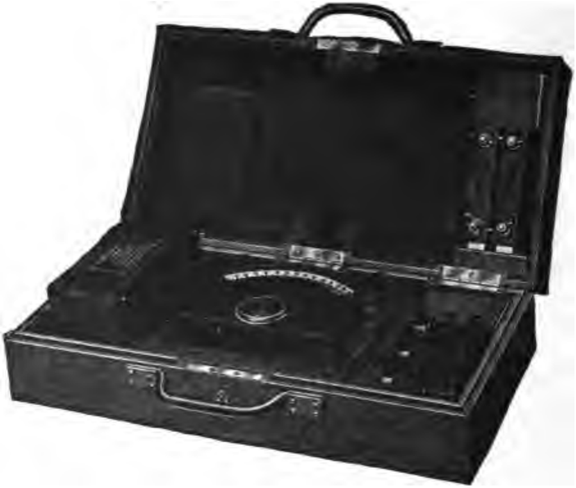


Figure 152.—Standard Direct-Current Ammeter and Voltmeter.

which may be calibrated by direct current and then used in the alternating-current circuit.

*Fundamental Principles of the Potentiometer.*—The potentiometer, in brief, is a special arrangement for measuring an electrical pressure by comparing it with the electromotive force of a standard cell. The operation of the potentiometer may be explained by means of the simplified diagrammatic drawing given in Figure 153. Assuming there is a constant pressure to be measured and that it is connected between the terminals $A+$ and $A-$, which are connected by a known resistance, say a slide wire of uniform cross-section. A second circuit, composed of a sensitive galvanometer *G*, a standard cell *SC*, a high resistance *H*, and a contact key *K*, is connected across a portion of the resistance between the points *A* + and *A* —, with the positive terminal of the standard cell toward *A* + and the negative terminal toward *A* —. The points of connections of this second circuit are so arranged that these may be moved along the main circuit and the portion of the main circuit between them varied. There will be a current produced in the slide-wire by the pressure between *A* + and *A* —, and, as a result, there will be a uniform drop in pressure for each unit length along the wire. Now, when the connections of the secondary circuit are so adjusted that there is no deflection of the galvanometer when the key *K* is closed, the electromotive force of the standard cell and the drops along the slide-wire between the points *C* and *D*, just balance each other. If the relation of the resistance or length of the section of the slide-wire *C*—*D* to the resistance or length of the entire wire is known, the total pressure between *A* + and *A* — may be calculated.

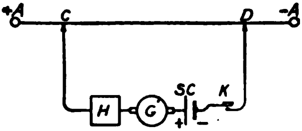


Figure 153.—Fundamental Principle of the Potentiometer.

The object of the high resistance *H* is to protect the galvanometer and standard cell when first trying for a balance, and it may be taken out of circuit when making a final or exact balance. The minimum electrical pressure it is possible to measure by the above procedure is one equal to the electromotive force of the standard cell. When pressures lower than this are to be measured, the operation is as follows: First, connect a steady source of electromotive force to the points $A+$ and $A-$ and measure its value by means of the standard cell as just described. Now substitute the pressure to be measured in place of the standard cell, observing the polarity, and obtain a second balance by adjusting the contacts $C$ and $D$. The value of this pressure can now be computed in terms of that between the points $A+$ and $A-$, which should be checked again to see that it has remained constant.

Ordinary resistance boxes may be used instead of the slide-wire as shown in Figure 153, but commercial forms of the potentiometer are on the market with all the resistances, keys, etc., contained in a single case.

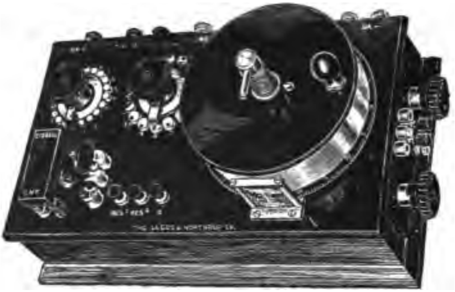


Figure 154.—Leeds and Northrup Potentiometer.

*Leeds and Northrup Potentiometer.*—A standard type of potentiometer in quite general use at the present time is shown in Figure 154, and a diagram of its electrical connections is given in Figure 155. The main circuit of the instrument consists of fifteen five-ohm coils, adjusted to equality, to a high degree of accuracy, connected in series between the points $A$ and $D$ and having in series with them an extended wire $DB$, the resistance of which from 0 to 1,000 on its scale (the entire scale reading from 0 to 1,100), is also five ohms. A contact point *M* is arranged so that it can make contact between any two of the five-ohm coils; and a contact point *M′* is arranged so that it can make contact at any point on the extended wire *DB*. Current from the battery *W* flows through these resistances and, by means of the regulating rheostat *R*, it is so adjusted to be exactly one-fiftieth of an ampere. The drop in potential across any one of the coils between the points *A* and *D* is consequently one-tenth of a volt and that across the extended wire is .11 volt. By placing the contact point *M′* at zero and by moving the contact *M*, the electrical pressure between the points *M* and *M′* may be varied by steps of one-tenth of a volt, from 0 up to 1.5 volts. The extended wire *DB* is divided into 1,100 equal parts. By moving the contact point *M′* from 0 to 1,100, the electrical pressure between the points *M* and *M′* may be varied by infinitely small fractions of a volt. To make use of this variable electrical pressure in making measurements, an unknown electrical pressure is introduced in series with a galvanometer between the points *M* and *M′* and in opposition to the drop in potential along the circuit from *A* to *B*. The contact points *M* and *M′* are then adjusted until the galvanometer indicates that there is no current through it, when

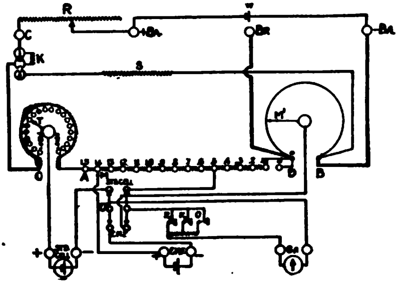


Figure 155.—Diagram of Potentiometer Connections.

the value of the unknown electrical pressure can be read from the position of the points *M* and *M′*. In the diagram, the connections show the unknown electrical pressure introduced at the binding posts marked E. M. F., the galvanometer at the posts marked *Ga*, and in series with these, three keys marked $R_1$, $R_2$, and $R_0$. When the key $R_1$ is depressed, the circuit is closed through a high resistance; when $R_2$ is closed, the circuit is closed through a low resistance; and when key $R_0$ is closed, the circuit is closed through zero resistance. The purpose of keys $R_1$ and $R_2$ is to protect the standard cell and the galvanometer in first trying for a balance.

At the point .5 on the series of resistances *A* to *D*, a lead wire is permanently attached which leads to one point of the double-throw switch marked *U*. Between the points *A* and *O* there is a series of 19 resistances with a sliding contact *T*. The resistance between the points *A* and the one marked .05 is exactly that which corresponds to an electrical pressure of one volt; and that between *A* and the point marked 1.0185 is a sufficient addition to make the resistance between the points .5 and 1.0185 correspond to a pressure of 1.0185 volts. Between the point marked 1.0185 and the end of the series of resistances there are 19 coils, which increase the corresponding electrical pressure by steps of .0001 to 1.0204 volts. This range corresponds with the variation in the different cadmium cells and the circuits are so connected that the two points *T* and .5 may be connected in series with the galvanometer and the keys $R_1$, $R_2$ and $R_0$. In this circuit are also the two binding posts marked *STD* cell, to which the standard cell is to be connected. To adjust the current to one-fiftieth of an ampere, throw the double-throw switch *U* to the upper position, as indicated by the dotted lines in the figure. Set the point *T* to correspond with the electromotive force of the standard cell and regulate the rheostat *R* until there is no deflection of the galvanometer when the keys $R_1$, $R_2$, and $R_0$ are closed, in the order given. The unknown pressure may then be measured as before with the contact points *M* and *M′*, the double-throw switch *U* having been placed in the lower position, as indicated by the full lines in the figure. After a balance has been obtained, the current in the main part of the potentiometer may be checked by simply changing the position of the switch *U* and closing the key $R_0$. A galvanometer balance shows that no change in current has occurred. A slight deflection calls for a slight readjustment of *R* and a corresponding readjustment of the point *M′*.

The resistance *S* is of such a value that when it shunts the circuit *0B*, the total resistance between the points 0 and *B* is one-tenth of the same unshunted. When the shunt is applied, provided the total current remains the same, the drop in potential between any two points on *AB* will be exactly one-tenth its previous value. The total current will remain the same, provided the total resistance of the circuit remains the same. This is accomplished by making the resistance of the coil *K* such that it exactly compensates for the reduction in resistance caused by plugging in the shunt coil *S*. The low scale is applied by moving the plug from the position 1 to .1. With this change the potentiometer reads from .16 of a volt down, by indicated steps of .000005 of a volt.

The use of the potentiometer in calibrating voltmeters and ammeters will be described in two of the following sections.

## CALIBRATING GALVANOMETERS

*Galvanometers.*—In general, the galvanometer is an instrument used in detecting or measuring small electric currents, or measuring the quantity of electricity discharged through a circuit, such as the charge or discharge of a condenser. When used in detecting or measuring an electric current, the galvanometer is called a *current galvanometer;* and when used in measuring a quantity of electricity, it is called a *ballistic galvanometer.*

*Constants of Current Galvanometer.*—The construction of the majority of galvanometers is such that the deflection of their moving system is practically proportional to the current flowing through the galvanometer winding, that is

$$I = K\theta$$

in which $I$ is the current in the winding; $\theta$ is the deflection of the moving system in any convenient unit; and $K$ is a constant, called the figure of merit of the instrument, which is defined as the current required to produce a unit deflection of the moving system. It is evident from the above that there will be as many different values for the figure of merit of a galvanometer as there are methods of measuring the unit deflection, and for this reason the unit in which the deflection is measured should always be specified when the value of $K$ is given.

A constant which is frequently used in expressing the sensibility of a galvanometer is the number of megohms (million ohms) that can be placed in series with the galvanometer to give one unit deflection when there is an electrical pressure of one volt acting on the circuit.

Another constant quite frequently used in expressing the voltage sensibility of a galvanometer is the electrical pressure required to produce one unit deflection.

Assuming the same method is employed in measuring the unit deflection—which is usually one millimeter on a scale one meter from the galvanometer mirror—when the three constants given above are determined, then the following relation will exist between them. The megohm sensibility is equal to the reciprocal of the figure of merit, and the voltage sensibility is equal to the product of the figure of merit and the resistance of the galvanometer.

The ballistic constant of a galvanometer is the quantity of electricity required to produce unit theoretical deflection. The actual deflection must be multiplied by the damping factor to get the theoretical deflection.

*Determining Figure of Merit of a Galvanometer.*—The scheme of connections of a simple method of determining the figure of merit of a galvanometer is given in Figure 156, in which $G$ represents the galvanometer; $K_1$ and $K_2$ are contact keys; $P$ is a potential rheostat; $V$ is a low-reading voltmeter; and $B$ is a battery of several cells. The electrical pressure acting on the galvanometer when its circuit is closed by operating the keys $K_1$ and $K_2$ is indicated by the voltmeter, and this pressure divided by the resistance of the galvanometer circuit gives the value of the current through the instrument. The observed deflection due to this current divided by the value of the current will give the value of the current per unit deflection or figure of merit, assuming the deflections are proportional to the current. The value of the deflection should be changed and the figure of merit re-calculated; and if the values for different deflec-

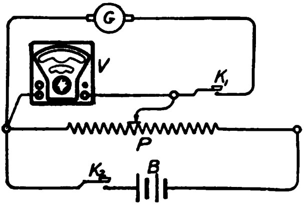


Figure 156.—Scheme of Connections for Determining Figure of Merit of a Galvanometer.

tions do not agree, a curve should be drawn indicating the relation between the figure of merit and the extreme deflection. When the galvanometer is used in measuring a current, the value of $K$, as determined from the curve for the particular deflection, should be used.

*Galvanometer Shunts.*—Frequently the current to be measured exceeds the capacity of the instrument, in which case a known resistance may be placed in parallel with the coil of the instrument and only a fractional part of the total current will pass through the galvanometer itself. Such a resistance is called a *galvanometer shunt*, and, if the relation of this resistance and that of the galvanometer is known, the total current in the main circuit may be calculated in terms of that passing through the galvanometer.

*Determining the Ballistic Constant of a Galvanometer.*—The ballistic constant of a galvanometer may be determined by sending a known quantity of electricity through its winding and observing the deflection, which, multiplied by the damping factor, gives the theoretical deflection. This theoretical deflection, divided into the quantity, gives the quantity per unit deflection, or the ballistic constant. A scheme of connections for determining the ballistic constant is shown in Figure 157, in which $G$ represents the galvanometer; $C$ is a standard condenser; $P$ is a potential rheostat; $V$ is a voltmeter; $S$ is a double-pole double-throw switch; and $K$ is a key to short-circuit the galvanometer. The standard condenser may be charged to a definite potential—determined by the adjustment of $P$ and indicated by $V$—by throwing $S$ into the lower position, and then discharged by throwing $S$ into the upper position. The quantity of electricity stored in the condenser when $S$ is down will be discharged through the galvanometer when $S$ is in the upper position and produce a deflection of the moving system. This deflection multiplied by the proper damping factor gives the theoretical throw of the moving system. The quantity of electricity in coulombs divided by the theoretical deflection gives the quantity per unit deflection, or the ballistic constant. The quantity of electricity stored in the condenser is given by the following equation:

$$Q = CE \times 10^{-6}$$

in which $Q$ is the quantity in coulombs; $C$ is the capacity in microfarads; and $E$ is the value of the pressure impressed on the terminals of the condenser in volts. The value of the constant for different extreme deflections should be determined, and if the values do not correspond, a curve should be drawn indicating the relation between the constant and the extreme deflection. When using the galvanometer in measuring quantity, the value of $K$, as determined from the curve for the particular deflection, should be used.

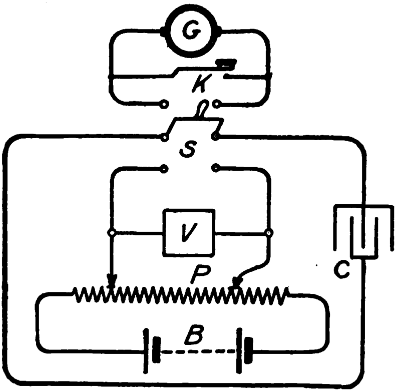


Figure 157.—Scheme of Connections for Determining Ballistic Constant.

*Damping Factor of the Ballistic Galvanometer.*— The actual deflection of the moving system of a ballistic galvanometer, due to the discharge of a certain quantity of electricity through its winding, is not as great as the theoretical deflection, nor is it the same for different resistances in series with the instrument. This difference between the actual and the theoretical deflections is due to what are called *damping forces,* which are due to the friction of the air on the moving system; friction of the support, if the moving system is supported; and the magnetic effect of induced currents. In using the ballistic galvanometer, the actual observed deflection should be multiplied by a factor called the *damping factor* in order to get the theoretical throw. The value of the damping factor is always greater than unity and its value should be determined under service conditions. A simple method of determining the numerical value of the damping factor is as follows: Set the moving system swinging, with a resistance in series with the galvanometer coil corresponding to service conditions, and observe the successive deflection of the moving system to the right or to the left of the zero position. The square root of the ratio of successive deflections is the damping factor for that average deflection and the particular resistance in series with the galvanometer.

## CALIBRATING VOLTMETERS

*Calibrating a Voltmeter by Comparison.*—The accuracy of a voltmeter's indication may be determined by connecting it in parallel with another instrument (called a standard) whose calibration has been accurately determined, and comparing their indications for various impressed voltages. The connections of the instruments are indicated in Figure 158, in which *S* represents the standard instrument; *X* is the instrument to be calibrated; *B* is a battery of desired pressure; and *P* is a suitable potential rheostat for varying the pressure over the two voltmeter terminals.

*Calibration of a Voltmeter by Means of a Potentiometer.*—When a greater degree of accuracy is desired in calibrating a voltmeter than can be obtained by the method of comparison or when it is necessary to check the standard instrument, the potentiometer is used. Pressures as high as 1.6 volts may be measured by connecting them to the terminals marked *E.M.F.* in Figure 155, and adjusting the points *M* and *M′*, a balance having previously been obtained with the standard cell, the resistance *K* being short-circuited throughout the test. Pressures between 1.6 and 16 volts may be measured by connecting them to the terminals marked *E.M.F.* and obtaining a balance by adjusting *M* and *M′* with the resistance *K* short-circuited, a balance having previously been obtained with the standard cell and the resistance *S* in circuit. The readings on the potentiometer, when a balance is obtained with unknown pressure, should be multiplied by ten.

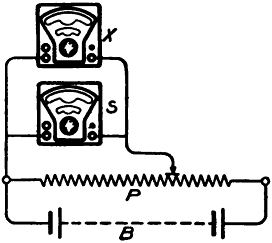


Figure 158.—Calibrating a Voltmeter by Comparison.

Pressures higher than 16 volts may be measured by connecting the pressure across a known resistance and measuring, by means of the potentiometer, the drop over a definite known part of this resistance. Knowing the ratio of the total resistance and the portion across which the drop is measured, the total pressure may be computed by multiplying the measured drop by the ratio of the resistances. Resistances suitable for this purpose are provided with the potentiometer previously described, and they are called *volt boxes,* or *multipliers.* The portion across which the drop is measured is usually a submultiple-of-ten part of the total resistance, and it is only necessary to multiply the potentiometer reading by the proper multiple of ten in order to obtain the true value of the total pressure.

*Alternating-Current Voltmeters.*—Alternating-current voltmeters may be calibrated by comparing them with a standard voltmeter of the electro-dynamometer or Kelvin balance type, which can be calibrated by means of direct current.

## CALIBRATION OF AMMETERS

*Calibrating Ammeters by Comparison.*—The ammeter may be calibrated by connecting it in series with a standard ammeter and comparing their indications for different values of current. The scheme of connections is shown in Figure 159, in which $S$ represents the standard ammeter; $X$ is the instrument to be calibrated; $B$ is a storage battery or other source of steady current; and $R$ is a current rheostat for adjusting the value of the current to the desired value.

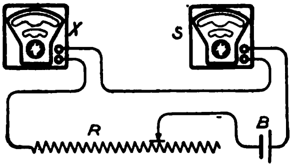


Figure 159.—Calibrating an Ammeter by Comparison.

*Calibration of an Ammeter by a Potentiometer.*—The potentiometer may be used in calibrating an ammeter by measuring the drop in potential over a known standard resistance connected in series with the ammeter. The current through the resistance is equal to the drop between its terminals in volts, as measured by the potentiometer, divided by the value of the resistance in ohms.

*Alternating-Current Ammeters.*—In calibrating alternating-current instruments, it is always best to connect them in series with a standard instrument of the electrodynamometer or Kelvin balance type and compare their indications when connected in an alternating-current circuit. The standard may be calibrated by direct current.

## CALIBRATION OF WATTMETERS

*Voltmeter-Ammeter Method of Calibrating a Wattmeter, Common Source of Current and Pressure.*—A voltmeter and an ammeter may be used in calibrating

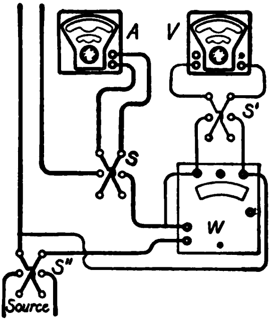


**Figure 160.—Voltmeter-Ammeter Method of Calibrating a Wattmeter, Common Source of Current and Pressure.**

a wattmeter operating on a direct-current circuit by connecting the ammeter in series with the current coil through a reversing switch *S* and the voltmeter in parallel with the pressure circuit through a reversing switch *S′*, as shown in Figure 160. From an inspection of the diagram, it will be observed that the current taken by the voltmeter passes through the series coil and, as a result, there is a greater current in the series coil than that indicated by the ammeter. The current taken by the voltmeter is usually quite small and may be neglected except in cases where it is an appreciable percentage of the load current in the series coil. The product of the current in the series coil and the pressure indicated by the voltmeter will give the true power; and the wattmeter indication, if correct, should correspond to this value. The current in the series coil and the pressure circuit of the wattmeter should be reversed by the switch $S''$ and a second reading taken for the same current and voltage to determine if there is any unbalance or to eliminate the effects of any stray magnetic fields. If there is any difference in the two wattmeter indications, the average of the two should be taken as the correct value. The instrument should be calibrated for currents and voltages corresponding to those of the circuit in which it is to be used.

*Voltmeter-Ammeter Method of Calibrating a Wattmeter, Separate Source of Current and Pressure.*—In calibrating a high-current capacity wattmeter, it is more economical to use a separate source of current and pressure, and it is also necessary to use a separate source in the calibration of a compensated wattmeter in order to determine if the compensation is correct. A scheme of connections for calibrating a high-current capacity wattmeter, using a separate source of current and pressure, is given in Figure 161. The battery connected in series with the current coil is of low-voltage and high-current capacity, while the one in series with the pressure coil is of high voltage and low-current capacity. No correction is needed for the current taken by the voltmeter in this case as it does not pass through the series coil, but the currents in the two circuits must be reversed and two readings of the wattmeter taken, as in the previous case.

The scheme of connections for calibrating a compensated wattmeter is the same as that given in Figure 161 except an additional switch $S''$ is required. The compensating turns may be connected in series with the pressure circuit by throwing the switch $S''$ to the left, and out of circuit by throwing $S''$ to the right. The difference in the readings of the wattmeter with and without the compensating turns in circuit should correspond to the loss in the pressure circuit for the particular pressure impressed between its terminals. The current in the circuits should be reversed and the average of the readings for the two directions taken as the correct value.

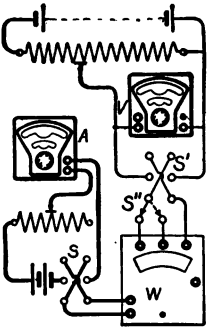


**Figure 161.**—Voltmeter-Ammeter Method of Calibrating a Wattmeter, Separate Source of Current and Pressure.

*Alternating-Current Wattmeters.*—Alternating-current wattmeters may be calibrated by connecting them in parallel with a standard wattmeter of the electrodynamometer or Kelvin balance type which may be calibrated on a direct-current circuit. Wattmeters used as standards should be carefully constructed and you should make sure that they are not affected by a phase displacement of the current and the pressure by having them checked at a reliable laboratory equipped for making such tests.

# CHAPTER XII

## TESTING WATT-HOUR METERS

*Classification of Tests.*—The meter committee of the National Electric Light Association recommends that the test on watt-hour meters be classified as follows:

| | |
|---|---|
| Shop tests | Inquiry tests |
| Installation tests | Re-tests |
| Periodic tests | Repair tests |
| Complaint tests | Special tests |

The *shop test* consists of a careful examination and test of all meters received from the manufacturer or those removed from service before they are placed in the stockroom or again installed.

The *installation test* is one made after the meter is installed to make sure that it is operating correctly. Very frequently the adjustment of a meter is changed or an accident may happen to it during installation and it is the purpose of the installation test to make the necessary adjustment if such should be required. Such a test is imperative in the case of the commutator type meters, as there is always a change in the friction of the brushes on the commutator until the meter is in good working condition. It is best to allow the meter to operate for several weeks before a final test is made, as the frictional losses will be fairly constant by that time. The induction type meters may be tested immediately after being installed, as their friction losses are practically constant, no commutator being required.

The *periodic test* is one made on a meter in service at more or less regular intervals in order to determine if its permissible error is exceeded. These periodic tests should be made at intervals to suit the circumstances; but no definite rule can be given for determining this interval of time for all classes of service, and meters of different capacities and types. It is customary to test commutator-type meters more frequently than induction or mercury-motor meters, as they are more likely to be in error. The higher the current capacity of the meter, the more frequent the test; and the tests are usually made more frequent in cases of large consumption than in cases where the average consumption is comparatively small, although the current capacities of the meters in the two cases may be the same.

A *complaint test* is one made on the meter of a certain customer who has complained of his bill. These tests are conducted in the same manner as the periodic test or the installation test, except the meter is usually tested on what is called its normal load or the load most generally used.

An *inquiry test* is one ordered by the company itself before rendering the bill, to determine if the meter has been operating properly. Such tests are usually made when a customer's bill for the current month differs a great deal from that of the preceding month.

A *re-test* is a test made on meters which have been opened or tampered with by any one not authorized; or if a meter has been out of service for some time, it should be tested when reconnected.

A *repair test* is one made on a meter which has been repaired while in service and the test should be made immediately after the repairs are completed.

*Special tests* are all tests which may not properly come under any one of the previous kinds, such as tests for interference, quarter phasing, determining meter torque, losses in the meter, etc.

*Meter Constants.*—The number of revolutions the moving element of a watt-hour meter makes in a given time is proportional to the energy that has passed through the meter during that time, and if the energy per revolution of the moving element is known, the total energy may be computed by multiplying the total number of revolutions by the value of the energy per revolution. This multiplication is usually performed automatically by the meter and the result is indicated on the dials of the instrument. In large capacity meters the difference between two successive readings of the dials seldom gives the value of the energy direct but this difference must be multiplied by a constant, which is usually marked on the dial, in order to get the value of the total energy. This constant is known as the *dial constant,* and it will have different values for meters of different capacities.

Most all meters have stamped or marked on them what is called the *test constant,* which is to be used in testing the meter, but there is, unfortunately, no uniformity in the meaning of the constant and its significance must be understood for each case. In some cases the value of the test constant corresponds to the number of watt-hours, watt-minutes, or watt-seconds per revolution of the disk, in which case it is called the *watt-hour, watt-minute,* or *watt-second constant.* If we represent these three constants by $K_h$, $K_m$, and $K_s$, respectively, then we may write the following relation:

$$K_s = 60\,K_m = 3{,}600\,K_h$$

In general the test constant has a value corresponding to one of the above, or a multiple of one of them.

*Relation of Test Constants for Different Watt-Hour Meters.*—In testing watt-hour meters, the accuracy of the meter is expressed by the ratio of the meter indication and the true energy, both in the same units, in per cent. This may be expressed in the form of an equation as follows:

$$\text{percentage of accuracy} = \frac{\text{meter energy}}{\text{actual energy}} \times 100$$

The numerator of the above equation is the energy indicated by the watt-hour meter in a given time, and the denominator is the actual energy passing through the meter as determined by the standard instruments.

The indication of the watt-hour meter being tested is usually determined by counting the number of revolutions of the disk in a given time and multiplying this number by the testing constant of the meter, or

$$\text{meter energy} = \text{testing constant} \times \text{revolutions}$$

Representing the revolutions of the meter disk in time $T$, in seconds by $R$, and assuming the test constant of the meter is the watt-seconds per revolution $K_s$, we may write the following equations:

$$\text{meter watt-seconds} = K_s \times R$$

or

$$\text{meter watts} \times T = K_s \times R$$

then

$$\text{meter watts} = \frac{K_s \times R}{T}$$

This formula, in some form, is used by the majority of meter manufacturers; and if they all used the formula just as it stands, the watt-second constant would be the test constant in every case. Different manufacturers, however, use the equation in different forms and, as a result, the simple relation just given does not hold in all cases. The constants as used by several of the leading companies and their relation to the watt-second constant of the meters are as follows:

Westinghouse Electric and Manufacturing Company:

$$\text{test constant } K_w = \text{watt-second constant}$$

$$K_s = K_w$$

$$\text{meter watts} = \frac{K_w \times R}{T}$$

General Electric Company:

$$\text{test constant } K_g = \text{watt-hour constant}$$

$$K_s = 3{,}600 \times K_g$$

$$\text{meter watts} = \frac{3{,}600 \times K_g \times R}{T}$$

Duncan Electric Manufacturing Company:

$$\text{test constant } K_d = \text{watt-hour constant}$$

$$K_s = 3{,}600 \times K_d$$

$$\text{meter watts} = \frac{3{,}600 \times K_d \times R}{T}$$

Fort Wayne Electric Works:
For "Type K" meters

$$\text{test constant } K_f = .01 \times \text{watt-second constant}$$

$$K_s = 100 \times K_f$$

$$\text{meter watts} = \frac{100 \times K_f \times R}{T}$$

For "Type $K_1$, $K_2$, $K_3$, and $K_4$" meters

$$\text{test constant} = \text{watt-hour constant}$$

$$K_s = 3{,}600\, K_f$$

$$\text{meter watts} = \frac{3{,}600 \times K_f \times R}{T}$$

Sangamo Electric Company:

$$\text{test constant } K_e = \text{watt-second constant}$$

$$K_s = K_e$$

$$\text{meter watts} = \frac{K_e \times R}{T}$$

*Example Illustrating the Use of the Test Constant.*—A General Electric 25-ampere two-wire D. C. 115-volt watt-hour meter with a watt-hour test constant equal to unity was tested by means of an ammeter and a voltmeter and gave the following results. Determine the accuracy of the meter.

*R*, revolutions of disk = 25
*T*, time in seconds = 42.2
Ammeter reading = 20 amperes
Voltmeter reading = 110 volts

*Solution.*—

$$\text{meter watts} = \frac{3{,}600 \times 25}{42.2} = 2{,}132.\text{—}$$

$$\text{Actual, or true watts} = 20 \times 110 = 2{,}200$$

$$\text{Percentage of accuracy} = \frac{2{,}132}{2{,}200} \times 100 = 96.9\%$$

*Methods of Testing Watt-Hour Meters.*—There are a number of different methods of testing service meters on the consumer's premises, and the following are probably the ones most generally used:

Ammeter-voltmeter method
Indicating wattmeter method
Portable rotating standard watt-hour meter method
House type watt-hour meter used as a standard method
Lamps or standard resistance method

*Ammeter-Voltmeter Method of Testing Watt-Hour Meters.*—Watt-hour meters may be tested by comparing the number of watts as shown by the meter under test with the number of standard watts determined by multiplying the readings of an ammeter and a voltmeter connected in circuit so as to indicate the current in the series coil of the watt-hour meter and the pressure acting on its pressure circuit, respectively. The ammeter and the voltmeter used should be of a reliable type, and they should be in good calibration or the true value of their indications should be known in order to accurately determine the standard watts.

The watts shown by the watt-hour meter are determined by timing a whole number of revolutions of the disk in seconds and fractions thereof by means of a reliable stop watch, and substituting the results in the following equation:

$$\text{meter watts} = \frac{K_s \times R}{T}$$

in which $K_s$ is the watt-second constant of the watt-hour meter; $R$ is the number of revolutions in $T$ seconds; and $T$ is the time of test in seconds. The duration of the test as usually taken is about 45 seconds.

The connections for testing a three-binding post two-wire, and a four-binding post three-wire watt-hour meter by means of an ammeter and a voltmeter are given in Figures 162 and 163, respectively. These connections may be modified to meet special conditions and the requirements of the meter being tested. For example, the pressure circuit of the three-wire meter in Figure 163 might be disconnected from the upper series coil within the meter and connected to the upper wire outside the meter, and the upper series coil then connected in series with the lower one, a

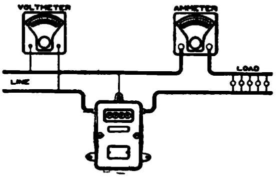


Figure 162.—Ammeter and Voltmeter Connections for Testing a Three-Binding-Post Two-Wire Watt-Hour Meter.

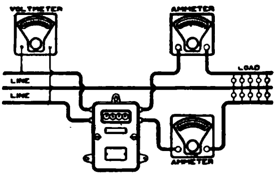


Figure 163.—Ammeter and Voltmeter Connections for Testing a Four-Binding-Post Three-Wire Watt-Hour Meter.

jumper being placed in the upper wire, and only one ammeter will be required. In testing watt-hour meters by this method, it is necessary to read both the ammeter and the voltmeter and in addition count the number of revolutions and operate the stop watch. Any variation in pressure or current will affect the final results; and when such changes are beyond the control of the tester, it is advisable to take a number of readings of both the ammeter and the voltmeter at regular intervals and then take the average of these as the value of the true current and pressure, respectively.

This method cannot be used in testing alternating-current watt-hour meters unless the current and the pressure are in phase, and it is not advisable to assume that such is the case even though the load is considered non-reactive.

In testing small capacity meters by this method, a calibrated resistance, which consists of a number of resistance units of different value with a suitable switching arrangement for varying the load, is sometimes used in obtaining the true or standard watts. The temperature coefficient of the material composing this resistance should be negligible and it should be calibrated in watts for different impressed pressures. When such a device is used, it is only necessary to read the voltmeter, and the true watts may be obtained from a table giving the calibrating results of the standard resistance. In this case an error in the voltmeter reading affects both the current and the pressure and, therefore, produces a greater error than if it affected the pressure alone.

*Indicating Wattmeter Method of Testing Watt-Hour Meters.*—An indicating wattmeter may be used in measuring the true or standard watts in testing a watt-hour meter in place of a voltmeter and an ammeter. The use of the indicating wattmeter simplifies the operation quite a bit, as one instrument takes the place of two, and its indication gives the standard watts directly without any calculation.

The method, however, is open to serious objections when using certain types of indicating wattmeters on direct current, due to the fact that these instruments, being designed for use on alternating-current circuits, are affected to a considerable extent by the earth's magnetism and other stray magnetic fields. This error may be practically eliminated by taking two readings of the indicating wattmeter with the current in the pressure and the series coils reversed in the second case, assuming the load current and the pressure remain the same. The average of the two wattmeter readings may be taken as the true watts.

The connections for testing a three-binding-post two-wire and a four-binding-post three-wire watt-hour meter by means of an indicating wattmeter are given in Figures 164 and 165, respectively. These connections may be modified to meet special conditions and the requirements of the meter being tested. For example, the two series coils of the meter in Figure 165 may be connected in series, as explained in the previous case, and a single wattmeter used instead of two. The calculations are the same as in the previous case.

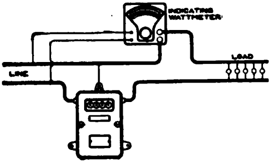


Figure 164. — Indicating Wattmeter Connections for Testing a Three-Binding-Post Two-Wire Watt-Hour Meter.

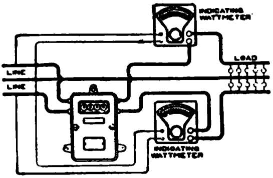


Figure 165. — Indicating Wattmeter Connections for Testing a Four-Binding-Post Three-Wire Watt-Hour Meter.

In testing alternating-current meters, it is frequently desirable to know the power factor of the circuit, and it may be determined by dividing the true watts as indicated on the wattmeter by the apparent watts, which can be determined by multiplying the indication of an ammeter in series with the series coil of the wattmeter by the indication of a voltmeter in parallel with the pressure coil of the meter.

*Portable Rotating Standard Watt-Hour Meter Method of Testing Watt-Hour Meters.*—The most reliable method, all things considered, in use at the present time for testing watt-hour meters and the one requiring the least time is the method employing the portable rotating standard. In this case the watt hours, as indicated by the watt-hour meter under test, are compared with the watt hours, as indicated by the rotating standard. The dial of the rotating standard is read at both the beginning and at the end of a certain number of revolutions of the watt-hour meter being tested, unless it is of the reset type, and the difference in these two readings gives the whole number of revolutions and fractions thereof made by the rotating standard. The percentage of accuracy of the meter may then be found by the following formula:

$$\text{percentage of accuracy} = \frac{K_s \times R}{K' \times R'} \times 100$$

in which $K_s$ is the watt-second constant of the meter under test; $R$ is the counted revolutions of the meter under test; $K'$ is the watt-second constant of the rotating standard; and $R'$ is the indicated revolution of the rotating standard. The rotating standard is usually started and stopped at the beginning and end of each test by operating a switch provided for this purpose.

In some of the earlier types of rotating standards for direct-current work, in which there is an error introduced due to the heating of the pressure coil, the meter is started and stopped by a switch in the current circuit, or the rotating element runs continuously and the indicating mechanism is engaged or disengaged by a magnet clutch whose winding is connected directly across the line and is opened or closed by means of a small switch. In alternating-current rotating standards, the meter is started and stopped by closing and opening the pressure circuit, since the error introduced due to the heating effect of the shunt current is not appreciable. This method is also employed in some of the direct-current rotating standards when the heating effect of the pressure circuit may be neglected. The switch controlling the meter is usually attached to the end of a piece of flexible duplex conductor long enough to allow the operator to control the rotating standard from a position near the meter being tested, so he may count the revolutions of the meter under test.

The connections for testing a three-binding-post two-wire meter and a four-binding-post three-wire meter by means of a Duncan rotating standard are given in Figures 166 and 167, respectively. These connections may be modified to meet special conditions and the requirements of the meter being tested.

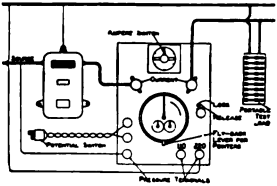


Figure 166.—Connections of a Rotating Standard for Testing a Three-Binding-Post Two-Wire Watt-Hour Meter.

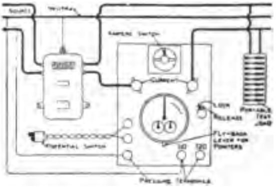


Figure 167.—Connections of a Rotating Standard for Testing a Four-Binding-Post Three-Wire Watt-Hour Meter.

*Rotating Standards.*—The fundamental principles upon which both the direct- and the alternating-current rotating standard watt-hour meters operate are the same as those of the service watt-hour meters, but their construction is somewhat different in order to meet certain requirements. The rotating standard must be a portable form of meter, which may be easily carried about by the meter tester; it must have a wide range of current-carrying capacities in order that it may be used in measuring full and light loads accurately and in testing meters of different capacities; and it must have an easy and a reliable means of determining the number of revolutions of the rotating element.

The first requirement of the rotating standard is met by dispensing with the iron case in which the watt-hour meter is usually placed, and enclosing all of the operating parts in a properly dimensioned wooden case with a hinged top and carrying handle or strap, as shown in Figure 168, which is a top view of a single-phase rotating portable standard made by the Westinghouse Company.

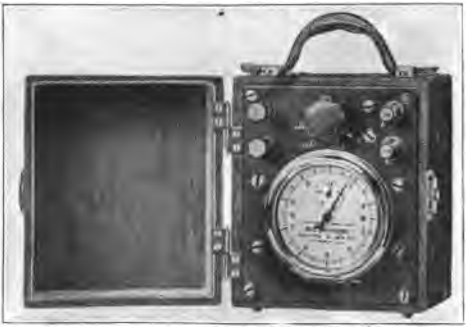


Figure 168.—Top View Westinghouse Single-Phase Portable Rotating Standard.

The second requirement of the rotating standard is met by making the current coils of the meter in sections, and mounting these sections so that they may be connected in series or in parallel by some kind of a special switching device. The torque acting on the rotating element is made the same for all the different full-load currents, as marked on the control switch on top of the meter, by making the effective ampere turns of the current coils the same at full load for all the different capacities.

The third requirement of the rotating standard is met by mounting a pointer on the upper end of the shaft of the rotating element. This pointer moves over a circular scale divided into 100 equal parts, and, with this arrangement, it is possible to read the one-hundredth part of a revolution. One or more smaller pointers indicate the whole number of revolutions. These pointers are plainly shown in Figure 168. A view of the rotating standard made by the Fort Wayne Electric Works, removed from the case, is shown in Figure 169.

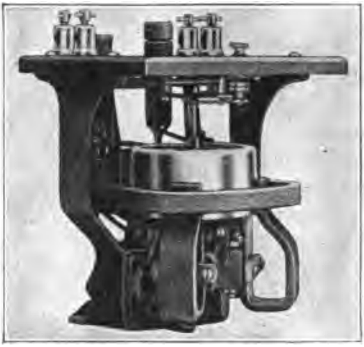


**Figure 169.—Rotating Standard of Fort Wayne Electric Works, Removed from Case.**

Rotating standards are built for polyphase testing, and they consist of two single-phase meters mounted on a common shaft and operating the same recording mechanism. These elements should be so located with respect to each other that there is no appreciable interference between them. It is quite necessary in using rotating standards, that they be placed where they are just as free from the effects of vibration as possible and at the same time not acted upon by any magnetic disturbances, such as motors, generators, or conductors in which there is a large current. Magnetic disturbances are more pronounced in the case of direct-current standards, and for this reason they should always be connected as indicated by the polarity signs at their terminals and placed in such a position that the plane of their current coils is in the direction indicated by a compass needle usually mounted in the top of the rotating standard.

*Testing Watt-Hour Meters with a House Type Watt-Hour Meter as a Standard.*—This method of testing a watt-hour meter consists in employing an extra carefully calibrated house type watt-hour meter as a standard, with special pressure binding posts arranged so the pressure circuit of the standard can be connected in ahead of the service meter. This connection is necessary in order that the standard meter will measure the true energy passing through the meter being tested.

The accuracy of the service meter is in some cases determined by connecting it in series with the standard watt-hour meter, and running the two on the consumer's load or on an artificial load for an hour, or day, or several days, reading the dial of each meter before and after the test. A comparison of the energy recorded by the two meters will give the percentage accuracy as follows:

$$\text{percentage of accuracy} = \frac{\text{service meter watt-hours}}{\text{standard meter watt-hours}} \times 100$$

A second method, which is much less tedious than the above, is to compare the revolutions of the two meters. The percentage of accuracy may be calculated by means of the following formula:

$$\text{percentage of accuracy} = \frac{K_s \times R}{K' \times R'} \times 100$$

in which $K_s$ is the watt-second constant of the meter under test; $K'$ is the watt-second constant of the standard meter; and $R$ and $R'$ are the revolutions of the meter being tested and the standard meter in any convenient interval of time.

The connections for testing a three-binding-post two-wire and a four-binding-post three-wire watt-hour meter by this method are given in Figures 170 and 171, respectively.

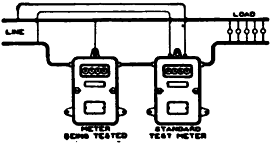


Figure 170.—Connections of a Standard Watt-Hour Meter for Testing a Three-Binding-Post Two-Wire Watt-Hour Meter.

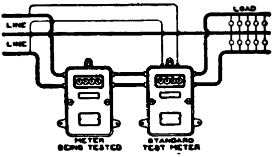


Figure 171.—Connections of a Standard Watt-Hour Meter for Testing a Four-Binding-Post Three-Wire Watt-Hour Meter.

*Testing Watt-Hour Meters with Lamps or Standard Resistances.*—In many plants where they have no standards at all, they are more or less handicapped in knowing just how accurately their watt-hour meters are recording and in such cases have to resort to the use of lamps of a given voltage in testing the meters. This method of testing is not recommended and is only used in the absence of the necessary instruments. If the wattage of the lamps used at different voltages is accurately known, quite accurate results may be obtained provided a voltmeter is available for measuring the pressure.

A similar method is sometimes employed where standard resistances are used in place of the lamps. These standard resistances are rated to take so many watts at a given voltage, and are more reliable than the lamps.

If no stop watch is available, it is more inconvenient to count the whole number of revolutions of the disk and estimate the fractional part of a revolution in a given time, than it is to estimate the time of a whole number of revolutions. The percentage of accuracy may then be determined by the following formula:

$$\text{percentage of accuracy} = \frac{K_m \times \text{r.p.m.}}{\text{load watts}} \times 100$$

in which $K_m$ is the watts-minute constant of the meter being tested; and r.p.m. represents the revolutions of the disk per minute.

*Testing Polyphase Watt-Hour Meters.*—Polyphase watt-hour meters consisting of two separate single-phase meter elements may be tested by connecting the current coils in series and the voltage coils in parallel, both combinations being connected to the same phase so that no error will be introduced due to improper phase relation of current and pressure.

Polyphase watt-hour meters may be tested under service conditions by loading the two elements separately when connected to two of the three phases. Two indicating wattmeters or a polyphase standard may be used in this case. When a polyphase meter is out of adjustment, it is advisable to test each of the elements separately, to determine if the two elements are properly balanced.

The rotating torque acting on the moving element of a polyphase watt-hour meter is the algebraic sum of the torques developed in each of the motor elements. These two torques are in the same direction, if the meter is connected correctly, when the power factor of the circuit is greater than .5; and in opposite directions when the power factor is less than .5, or the current lags the pressure by more than 60 degrees. When transformers are used in combination with watt-hour meters, it is rather difficult to determine by inspection whether the meter elements are properly connected, in which case the following test will be useful. Disconnect one of the elements, by bridging the secondary of the current transformer and then disconnecting the leads to one element of the meter, and note the direction in which the meter rotates due to the torque developed in the connected element. Re-connect this element and disconnect the other one and note the direction of rotation. If the load is a lamp load, the power factor is greater than .5 and both motor elements should cause the meter to revolve in the same direction; but, if the test shows they do not, the connections of one element must be changed. If the load is an induction motor or transformer, the power factor may be greater or less than .5 depending upon the load on the motor or transformers. When the motor or transformers are operating without load or very light loads, the power factor of the circuit is less than .5 and for loads near full-load the power factor is greater than .5. If the direction of rotation of either of the elements of the meter when acting alone changes when the load on the motor is increased, the light-load power factor is less than .5, and the elements should produce rotation in the opposite directions on light loads. When there is any doubt in your mind as to the power factor of the circuit, it is best to disconnect the connected load and try each element separately with a few lamps as a load on the phase in which the element being tested is connected.

*Methods of Loading Watt-Hour Meters for Test.*—There are a number of different methods employed in loading watt-hour meters under test, the more common of which are as follows:

The consumer's load
Portable group of lamps
Special testing rheostats
Portable storage batteries
Step-down transformers

The meter tester does not need to carry as much equipment when the consumer's load is used in testing watt-hour meters, but the companies usually prefer some other method on account of the annoyance to the consumer and the likelihood of their misunderstanding the object of the test.

A number of lamps mounted on a convenient base for carrying and provided with a suitable switching device so the number of lamps in the circuit may be changed is frequently used in testing watt-hour meters of low-current capacity.

There are a number of specially constructed rheostats on the market at the present time, composed of a number of resistances, of different value, connected in parallel by special switches. The wattage consumption of these resistances is given for different voltages and the only instrument required in determining the true watts is a voltmeter.

In testing direct-current meters, the desired current in the current-coils may be obtained from a low-voltage portable storage battery, which results in a considerable saving in energy for making the test. The value of the current may be easily regulated by means of a carbon-plate current rheostat connected in series with the battery and the current coils. The pressure circuit of the meter should have full or line pressure connected to its terminals.

The low-voltage transformer can be used in producing a large current in the current coil of the alternating-current watt-hour meter without taking a large current from the line, which results in a saving in energy. When such transformers are used, an accurate test should be made to determine their effect upon the power factor so that proper allowance can be made.

*General Procedure in Testing Watt-Hour Meters.*—The following method of procedure may be followed in testing watt-hour meters, which will result in all testers making the same tests and more uniform results will be obtained:

*First,* determine if the frame of the meter is grounded or crossed with a line whose potential is different than the ground. When such a cross is found, special precaution should be taken in removing the cover and making the necessary connections for the test.

*Second,* place jumpers around the meter to take care of the customer's load, unless this load is to be used in testing the meter.

*Third,* determine if there is any creep when the meter is connected to the line and there is no current in the current coils. If there is any creep, determine the time in seconds for one revolution of the disk, and make a record of this time.

*Fourth,* connect the testing instruments and load, and obtain test readings for at least heavy load and light load, and it is also best to make a test on normal load. The results of these tests should be carefully recorded.

*Fifth,* give the watt-hour meter a thorough cleaning; remove the register and inspect same to see that it is free from excessive friction; examine the commutator and brushes, if the meter is of the commutator type, and carefully clean and adjust them; clean the upper end of the shaft and worm gear, also the upper bearing; carefully inspect and clean the lower jewel and pivot, if such are used in the meter, and renew them when found defective; clean all dirt from between the magnets and adjust the disk to the proper height; replace the register, making sure it properly meshes with the worm on the shaft, and that there is sufficient vertical play for the moving system.

*Sixth,* after the meter has been cleaned and adjusted as described above, the friction compensation should be carefully adjusted so there is no creep and the meter will register on very light loads. Creeping due to fluctuations in voltage or slight vibration may be prevented in direct-current meters by placing a small metal clip of iron on the edge of the damping disk; and in the case of alternating-current meters, by drilling small holes in the disk.

*Seventh,* re-calibrate the meter on both light and heavy loads, and make the necessary adjustments. After all the adjustments have been made, re-check the meter so as to verify its accuracy.

*Eighth,* connect the consumer's load to the meter, remove the jumpers, and make a thorough inspection of all connections and the operation of the meter, making sure the disk rotates in the proper direction.

*Ninth,* seal the meter and make out a complete report.

In checking alternating-current watt-hour meters the cover should be in position as it will affect the magnet fields produced by the pressure and series coils and, as a result, the readings will be different. This difference is not as noticeable in the case of direct-current watt-hour meters.

*Causes of Inaccurate Watt-Hour Meter Readings.*—If any watt-hour meter is found in error, it is always advisable to try and locate the cause of the inaccuracy rather than make a compensation for the cause of the trouble. The principal causes of inaccuracies in watt-hour meter readings are as follows, and the meter tester should satisfy himself that such causes do not exist before any adjustments are made.

Defective armature
Defective series resistance
Improper adjustment of brushes
Defective jewels

The operation of a watt-hour meter is frequently affected by a defective armature, one or more of the coils being open or shorted. A voltmeter may be used in testing for opens and shorts by measuring the drop over each of the coils and comparing this drop with the difference in pressure between the brushes. The drop in pressure over each coil should be approximately equal to the drop between the brushes divided by twice the number of coils on the armature. An open coil will show a high drop and a shorted coil a low drop. An incandescent lamp may be used in making this test, and it will burn brighter when connected to an open coil and less so when connected to a shorted coil than it does when connected to a perfect coil. Two pointed metal terminals may be used in connecting the lamp to the terminals of the coils, and the voltage rating of the lamp should be something near the normal drop over the coils.

When the resistance in series with the pressure circuit of a watt-hour meter is open, the meter will not operate; and when a portion of it is short-circuited, there will be more than normal current in the pressure circuit and, as a result, the meter will run too fast. The different sections of this resistance may be tested for opens and shorts by means of a voltmeter or suitable test lamp.

If the tension of the brushes on the commutator is properly adjusted, there should be no appreciable wear and the friction between the brushes and the commutator will become practically constant after the meter has been in operation for a few weeks. The surface of the commutator often becomes roughened due to sparking, which may be caused by vibration, dust or dirt, or improper brush tension. If the commutator appears smooth and there is some sparking at the brushes, the trouble may be due to a dirty commutator or too light a brush tension. The surface of the commutator should be cleaned with a narrow strip of linen tape and any dirt between the segments removed with a finely pointed tool and the brush tension increased. When the surface of the commutator is roughened, it should be smoothed down with a piece of crocus cloth which has been worn practically smooth, and polished with a piece of linen tape. The surface of the brushes in contact with the commutator should be cleaned and polished, as in the case of the commutator, and their tension carefully adjusted so that they both bear the same amount on the commutator.

A defective jewel or pivot may cause excessive friction and, as a result, the meter will run slow. They should both be removed from the meter and examined with a small magnifying glass. If there is no defect in either, they should be thoroughly cleaned with a piece of very soft wood and replaced. A jewel may be examined for defects by moving a fine pointed needle back and forth over its surface, the needle being held loosely between the fingers. A very small quantity of jeweler's watch oil should be applied to the jewel bearing before it is replaced.

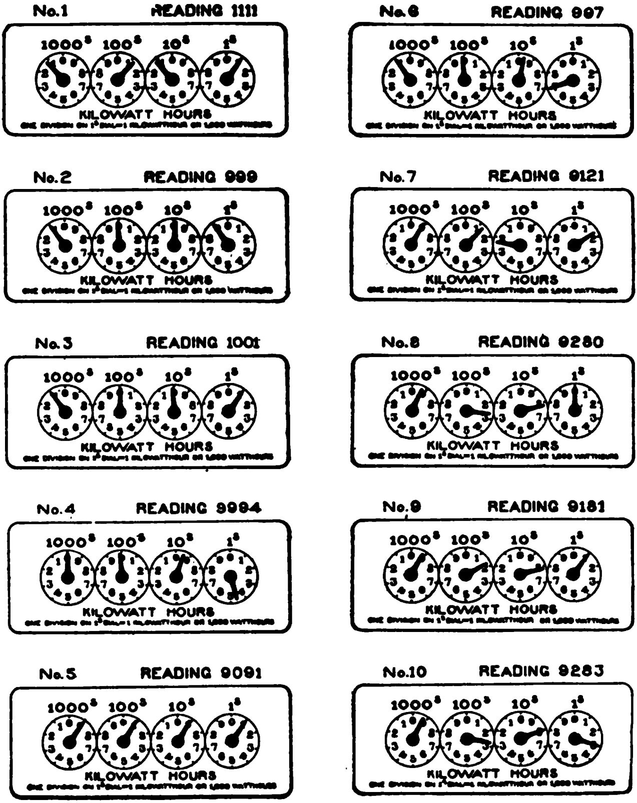
*Meter Records.*—A complete record of all meters should be carefully kept by all operating companies of any size in order that all information relative to any particular meter may be easily obtainable. These records are usually of the card-index type, as experience has proved they are better adapted for this kind of work than any other method. These records should give all the information relative to purchase, type, installation, constants, nature of load, readings, tests, and any other information which will be of service to the company. These records are not, as a rule, all kept on a single card; but usually several cards are used, the classification being based on the nature of the recorded information.

*Special Watt-Hour Meter Tests.*—Special tests are often made on watt-hour meters to determine their torque; the effect of stray magnetic fields upon their operation; the watts lost in the pressure and current coils; the interference of different metering elements; the effect of a change in the power factor or the frequency of the line; etc. These tests usually require special equipment and are as a rule conducted in laboratories equipped for such work. The results of all of the above tests are of importance in the operation of the meter and should be considered in the purchase of watt-hour meters.

*Reading Watt-Hour Meters.*—It is very essential that all watt-hour meters belonging to a central station company be accurately read in order to avoid trouble in over- or under-charging the consumers. It is customary with the larger companies to have a certain number of men who do nothing but read meters, a portion of the total number of meters being taken care of each day.

It is a very simple matter to read a watt-hour meter when the following general instructions are observed. The pointers on the different dials of watt-hour meters are geared to each other in such a way that a complete rotation of any pointer will cause the pointer to its left to move through one-tenth of a revolution. Pointers on adjacent dials rotate in opposite directions. The right-hand dial usually indicates units, and the second, third, fourth, etc., dials will then indicate tens, hundreds, and thousands, etc., respectively.

The reading of any dial should always be taken as the number on the dial which the pointer has last passed unless it is very near the number, in which case reference must be made to the dial to its right or next lower dial. When the pointer on the next lower dial has not completed a revolution, the figure in doubt on the upper dial should not be read but the next lower one; but, if the pointer on the lower dial


No.1 READING 1111

No.2 READING 999

No.3 READING 1001

No.4 READING 9994

No.5 READING 9091

No.6 READING 997

No.7 READING 9121

No.8 READING 9280

No.9 READING 9181

No.10 READING 9283

Figure 172.—Ten Typical Watt-Hour Meter Readings.

has completed a revolution, the figure in question on the upper dial should be read. On account of the above relation in the dial readings, the men reading the meters frequently read the dials from right to left rather than from left to right. After the readings of the dials have been taken and checked, the result should be correctly pointed off, as the dials on different meters do not always read in the same units. If a dial constant is required in computing the actual reading, it should be marked on the record. Ten typical watt-hour meter readings are given in Figure 172.

# CHAPTER XIII

## SPECIAL INDICATING AND RECORDING INSTRUMENTS

### POWER FACTOR METERS

*Purpose of Power Factor Meters.*—The power factor of an alternating-current circuit has been defined as the cosine of the angular displacement between the current and the electrical pressure, or as the quotient obtained by dividing the true power, as indicated by a wattmeter, by the apparent power, obtained by multiplying the value of the effective current by the value of the effective pressure as indicated by an ammeter and a voltmeter, respectively. The value of the power factor of any circuit determined by either of these methods will be the same, provided the current and the pressure are true sine waves. When the wave forms do not follow a sine law and especially when the current and the pressure curves have different forms, the power factor as determined by the two methods will not be the same. In all practical work the power factor is determined by dividing the true power by the apparent power, and may be determined as indicated above by means of a voltmeter, ammeter, and wattmeter. This method requires a calculation in each case, and for this reason a separate instrument has been developed, called a *power-factor meter,* which indicates directly the value of the power factor.

*Electrodynamometer Type Power-Factor Meter.*—The electrodynamometer power-factor meter is very similar in construction to the electrodynamometer type wattmeter, except the power-factor meter is provided with two movable coils in place of one, as in the wattmeter, and there are no controlling springs. The movable coils are mounted at right angles to each other and the electrical connections are made through springs which exert practically no torque. A non-inductive resistance is connected in series with one of the movable coils and a high inductance in series with the other, which results in the current in one coil, say coil *A*, being in phase with the pressure between its terminals, and the current in the other coil, or coil *B*, being displaced in phase from the pressure between its terminals by about 90 degrees. These two circuits are connected in parallel and constitute the pressure circuit of the instrument. The stationary coils are connected in series and carry the load current of the line in which the meter is connected, unless current transformers are used. If the line pressure and the load current are in phase, the current in the movable coil *A* will be in phase with the current in the stationary coil and the current in the movable coil *B* will be displaced in phase from the current in the stationary coil by about 90 degrees. This phase relation of the pressure and series currents results in there being a maximum action between coil *A* and the stationary coil and practically zero action between coil *B* and the stationary coil, and, as a result, the moving system will set itself in such a position that coil *A* is parallel to the stationary coils. When the phase relation between the load current and the line pressure changes, there will be a change in phase relation between the currents in the pressure coils and the current in the series coils, which will result in the resultant magnetic action on the movable coil *B* increasing and the action on coil *A* decreasing, and the moving system will turn into a new position, which will be a measure of the power factor of the circuit in which it is connected. The deflection of the moving system will be in opposite directions for leading and lagging currents. A pointer attached to the movable element will indicate the power factor on a properly calibrated scale.

The front view of a three-phase three-wire power-factor meter of this type, with the cover removed, manufactured by the Weston Electrical Instrument Company, is shown in Figure 173.

*Induction Type Power-Factor Meter.*—The induction type power-factor meter operates on the principle of a rotating magnetic field. This rotating magnetic field, in the case of a three-phase meter, is produced by three coils, whose planes make an angle of 60 degrees with each and carrying load currents displaced in phase by approximately 120 degrees. In the two-phase meter, the coils are at right angles to each other and the load currents in them are displaced in phase by approximately 90 degrees. A specially shaped iron vane is mounted on a shaft within this rotating magnetic field so that it is free to turn. The axis of this shaft is perpendicular to the plane containing the axis of the current coils. A coil, carrying a current in phase with the pressure, is placed about the iron vane with its axis parallel to the shaft on which the vane is mounted. This coil will magnetize the vane to a degree and polarity determined by the value of the current in the coil and its direction,

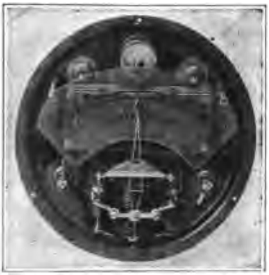


Figure 173.—Front View of Three-Phase Three-Wire Weston Power-Factor Meter.

respectively. The vane will then move into such a position that its magnetism passes through zero value at the same time the rotating field at that particular point is zero, and its position will be an indication of the power factor of the circuit.

In the single-phase meter, the relative position of the current and the pressure circuits are reversed, the rotating magnetic field being produced by two pressure coils in parallel and carrying currents which are displaced in phase by approximately 90 degrees. A more detailed description of an instrument of this type is given in the section of this chapter devoted to synchroscopes.

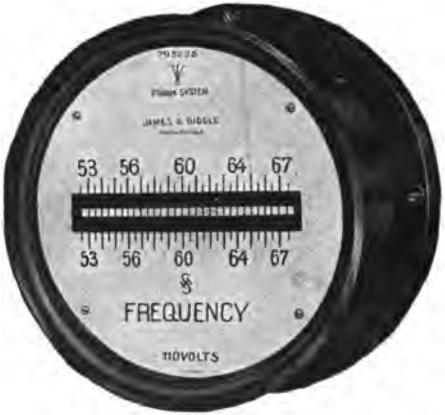


Figure 174.—Resonance Frequency Meter.

## FREQUENCY METERS

*Purpose of Frequency Meters.*—The frequency meter is an instrument designed to indicate the frequency of a circuit to which it is connected. The frequency of an alternating current, or pressure, is defined as the number of cycles per second, in which the cycle consists of a complete set of positive and negative values. The number of cycles per revolution, in the case of an alternator, is equal to one-half the number of poles $P$, and if the machine runs at $S$ revolutions per second, the cycles per second will be given by the formula

$$f=\frac{P\times S}{2}$$

*Resonance Frequency Meter.*—The resonance frequency meter consists of a number of steel reeds, having a different natural period of vibration, placed in a magnetic field produced by a current whose frequency is to be determined. When the magnetic action of the field corresponds to the natural period of any one of the reeds, it will cause that particular reed to vibrate, just as a note played on a violin may cause some particular string on a piano to vibrate. The ends of these reeds are so arranged that they vibrate in a rectangular opening cut in the face of the meter with a suitable scale marked along the edge of the opening. A front view of a frequency meter of this type is shown in Figure 174.

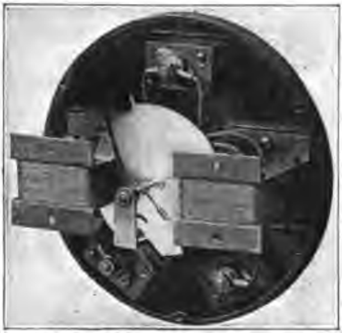


**Figure 175.—Westinghouse Induction Type Frequency Meter.**

*Induction Type Frequency Meter.*—A frequency meter of the induction type as manufactured by the Westinghouse Electric and Manufacturing Company is shown in Figure 175, with the cover removed. The meter consists of two induction type voltmeter elements, acting on a specially shaped disk which constitutes the moving element, and tending to move the disk in opposite directions. One of the voltmeter elements is in series with a non-inductive resistance and the other is in series with a reactance, so that a change in frequency tends to change the relative strength of the two magnets and cause rotation. The disk is so shaped that, as it moves, the amount of metal under the stronger magnet becomes less than that under the weaker magnet, so that with every relation between the two magnet strengths there is a point where the two torques produced by the two magnets balance and the moving system comes to rest. A change in line pressure produces the same effect on both coils and hence no change in the meter's indication.

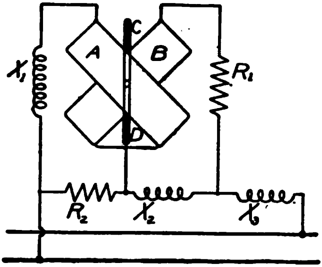


Figure 176.—Connections of the Stationary Elements of the Weston Frequency Meter.

*Weston Frequency Meter.*—The Weston frequency meter is somewhat similar in construction to the movable core type of ammeters and voltmeters. The magnetic field in the case of the frequency meter remains constant so long as there is no change in the pressure acting on the meter, but its direction changes with a change in frequency. The magnetic field in the frequency meter is produced by currents in two coils which are mounted at right angles to each other. A scheme of connections of the two coils is given in Figure 176. The two coils $A$ and $B$ are connected in series across the line with an inductive reactance $X_1$ in series with coil $A$ and a resistance $R_1$ in series with coil $B$. A resistance $R_2$ is connected in parallel with $X_1$ and $A$, and a reactance $X_2$ in parallel with $R_1$ and $B$; then the whole combination is connected in series with a reactance $X_3$, the purpose of which is to damp out the higher harmonics. The circuits, shown in Figure 176, form a Wheatstone bridge, which is balanced at normal frequency. An increase in frequency will increase the effect of the reactors $X_1$, $X_2$, $X_3$, and thus change the balance of the bridge, causing more current to flow through one coil and less current through the other. Any change in frequency is accompanied by a corresponding change in the direction of the resultant magnetic field.

The soft iron core $CD$ mounted on a shaft in the center of the coils takes up a position determined by the resultant magnetic field; and its position is a measure of the frequency of the circuit to which the meter is connected. A pointer attached to the moving system moves over a calibrated scale and gives a direct reading of the frequency.

## SYNCHROSCOPES

*Purpose of Synchroscopes.*—A synchroscope, or synchronism indicator, is a device for determining whether two synchronous machines are in synchronism, that is, whether their frequencies are the same or not, and whether the terminals of the switch for connecting the machines are of the proper polarity.

Perhaps the simplest device for this purpose is an ordinary incandescent lamp or a combination of lamps connected between the terminals of the machines, as shown in Figure 177. When there is no difference in potential between the terminals of the switches to which the lamps *L* are connected, no current will flow through the lamps, consequently they will not light up. The terminals of a given lamp will be at the same potential when the electromotive force of the two machines are equal and exactly opposed with respect to each other. This device is not altogether satisfactory as it requires a considerable pressure between the terminals of the lamps before it begins to light up, and the electromotive force of the two machines may be quite different without showing on the lamps. If the lamps be connected diagonally across the machines, they will burn at full voltage when the electromotive force of the machines are in synchronism and exactly opposed to each other, as measured between the terminals which are to be connected together at the switch. In either case the maximum pressure which may act on the lamps is equal to the combined electromotive forces of the two machines, and the lamps should be chosen accordingly. A number of lamps may be connected in series for higher voltages.

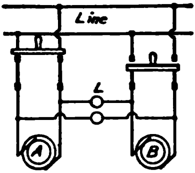


Figure 177.—Connection of Lamps for Synchronizing.

Modern synchroscopes are replacing the lamps, as they indicate the difference in speeds of the two machines—which one of the machines is running the faster—and also the phase difference between the two when they are running in synchronism. The operating principle of synchroscopes is practically the same as that of the power-factor meters.

*Weston Synchroscope.*—A diagram of the connections of the Weston synchroscope is shown in Figure 178. The two fixed coils $M_1$ and $M_2$ are connected in series to the terminals of the running machine *B* and carry a current in phase with the pressure of the machine, while the movable coil $M_3$ is connected to the terminals of the incoming machine *A*. The current through the stationary and the movable coils are displaced in phase from each other by 90 degrees when the electrical pressures acting on them are either in phase or phase opposition. This phase relation of the currents is brought about by means of a condenser *C* connected in series with the movable coil and the slight inductance possessed by the fixed coils. When the currents are displaced in phase by exactly 90 degrees, the movable coil will come to rest at right angles to the stationary coils and there will be no tendency for it to turn. If the phase displacement of the currents becomes greater or less than 90 degrees due to the pressures acting on the circuits being out of phase or phase opposition, there will be a torque acting on the movable coil tending to turn it from its zero position. This torque causes the movable coil to turn against the action of a spiral spring which holds the movable coil normally at right angles to the stationary coils. The torque acting on the movable coil becomes a maximum when the currents in the coils

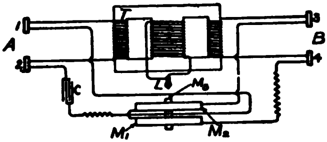


Figure 178.—Connections of the Weston Synchroscope.

are in phase or phase opposition. The direction of the torque will depend upon the relative direction of the currents in the coils; that is, the direction of the deflection indicates whether one lags or leads with respect to the other.

If the frequency of the pressures acting on the movable and the stationary coils are not the same, the current in one coil will be continuously changing its phase relation with respect to the current in the other coil, and, as a result, the torque acting on the movable coil will increase from zero to a maximum in one direction and back to zero, increase to a maximum in the opposite direction and back to zero, while the frequency of one of the pressures is increasing or decreasing one cycle with respect to the frequency of the other pressure. When the pressures have the same frequency but are not in phase, the movable coil will come to rest at one side or the other of the zero position, which will be determined by the average torque and the strength of the controlling spring. The instruments are so connected that the pointer attached to the movable coil is deflected to the left of its zero position when the incoming machine is slow, and to the right when the incoming machine is running too fast.

The synchronizing lamp *L* is connected to the secondary of a special transformer *T*, which is provided with two primary windings. One primary winding is connected to machine *A* and the other to machine *B*. When the terminals marked *1* and *3* are of the same polarity, there is a maximum magnetic flux through the secondary winding and the lamp is brightest at this time, which corresponds to exact synchronism of the two machines.

*Westinghouse Synchroscope.*—One type of synchroscope manufactured by the Westinghouse Electrical Manufacturing Company is practically the same as the power-factor meter. made by the same Company and previously described in this chapter. A rotating magnetic field is produced by the currents in two coils, *M* and *N*, Figure 179, connected to the bus bars, one through a reactor *P*, and the other through a resistor *L*. A movable iron vane or armature *A* is mounted on a shaft within the rotating magnetic field, the axis of the shaft being perpendicular to the plane containing the axis of the two coils *M* and *N*. This iron vane is magnetized by a coil *C* connected across the terminals of the incoming machine. The axis of the coil *C* corresponds in position to the shaft upon which the vane *A* is mounted. The iron vane is magnetized by the coil *C* and will be attracted or repelled by the rotating magnetic field of the coils *M* and *N* and will tend to take up a position such that the zero of the rotating field occurs at the same instant as the zero of its own field. Thus the position of the vane at any time indicates the phase angle between the pressure of the busbars and the pressure of the incoming machine. The iron vane with the pointer attached to it rotates, as the phase difference between the two pressures changes; and when the same frequency is reached, it remains stationary. Speeds of the incoming machine below synchronism cause the pointer to rotate in one direction marked "slow" on the dial and

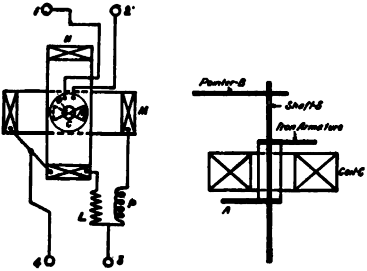


**Figure 179.—Diagram of Westinghouse Type TI Synchroscope.**

speeds of the incoming machine above synchronism cause the pointer to rotate in the opposite direction marked "fast" on the dial. The rate of rotation will depend upon the difference in frequency of the two pressures.

A diagram of the internal connections of another type of synchroscope made by the Westinghouse Electric and Manufacturing Company is shown in Figure 180. A bipolar laminated field *M* has its winding connected to the busbars, and hence to the machine in operation. Two coils *B* and *C* are wound at right angles to each other on an iron core *D*, which is mounted on a shaft so that the core and coils can freely rotate. Connections are made to these two coils by means of three slip rings and three brushes. A non-inductive resistance *R* is connected in series with one coil and an inductive resistance *L* in series with the other. The currents in the two coils are displaced in phase, by approximately 90 degrees, and, as a result, produce a rotating magnetic field. If the frequency of the currents in the coils *B* and *C* is the same as that of the current in the coil about the magnets *M*, there will be no tendency for the core *D* to rotate. Any difference in frequency of the currents in the

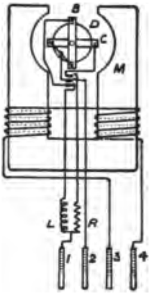


**Figure 180.—Diagram of Westinghouse Types C and D Synchroscopes.**

stationary and movable coils will cause a rotating of the core $D$ in one direction when the incoming machine is too fast, and in the opposite direction when the incoming machine is too slow. The rate of rotation will depend upon the difference in frequency of the two sources of current.

## RECORDING OR GRAPHIC INSTRUMENTS

*Purpose of Recording Instruments.*—A recording instrument is one which records on a time-chart a continuous record of the value of the quantity the instrument is supposed to measure. Recording instruments are constructed to give a continuous indication of the successive instantaneous values of all quantities whose values may be given by an indicating instrument, such as current, pressure, power, frequency, power factor, etc.

A complete knowledge of the fluctuations in electrical quantities is of great importance to the central station company in making a complete study of the operating characteristic of the station. Such instruments are also useful to the consumer inasmuch as they may give useful information relative to the operation of certain machines or departments.

*Classification of Recording Instruments.*—The recording, or graphic, instrument, as described in the following sections, may be divided into two general classes: the direct-acting type and the relay type.

In the *direct-acting type,* the moving element of the meter carries the pen or tracing point and causes it to move over a properly ruled chart which is driven at a constant speed by a suitable clockwork.

In the *relay type* of recording meter, the moving element of the meter proper operates a set of contacts, which close an auxiliary circuit. This auxiliary circuit contains the solenoids which operate the pen. In the relay type, the acuracy of the indications does not change with a change in friction of the pen upon the records, and the instrument may, as a rule, be adjusted to respond to a smaller variation in the quantity being measured than in the direct-acting type.

A second classification of recording instruments may be made, according to the form of the record obtained, into circular-chart instruments, which may give a record for a few days; and into continuous-chart instruments, which give a continuous record for as much as two months, depending upon the speed at which the record paper is drawn under the pen.

*Bristol Recording Ammeters, Voltmeters, and Wattmeters.*—A Bristol recording ammeter, with the cover removed, is shown in Figure 181. The current coil *C* is entirely stationary, and is attracted to the instrument back. The moving element consists of a combination disk armature *D* and light iron core mounted on a non-magnetic shaft extending through the current solenoid. Both ends of this shaft are supported by vertical steel springs *S* and $S_1$, and the right-hand spring is attached to the pen arm *A*. The armature core is normally held away from the coil *C;* but it is attracted and drawn toward the left-hand end of the coil when there is any current in the coil. Any slight

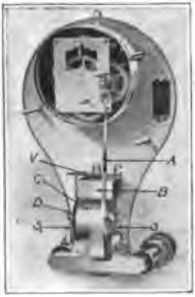


Figure 181.—Bristol Recording Ammeter.

movement of the core causes the upper end of the pen arm to move through a comparatively large angle.

The voltmeter consists of a stationary and a movable coil connected in series. The movable coil is mounted on a shaft and takes the place of the disk in the case of the ammeter. This shaft is supported at the ends on vertical springs and the pen arm is attached to the right-hand spring. The two coils are wound to repel each other, making them non-inductive with respect to each other.

A Bristol recording wattmeter, with the cover removed, is shown in Figure 182. The stationary coil *C* is wound with heavy wire capable of carrying the total current in the line, and the movable coil *P* is wound with fine wire and mounted on a shaft supported on vertical springs. The movement of the movable coil is proportional to the combined magnetic actions of the load current and a current which varies with the line pressure, or it is proportional to the power. The motion of the movable coil is imparted to the pen arm and causes the upper end of the arm to move through a relatively large distance. The action of the movable coil in producing a movement of the pen arm is multiplied by the use of a double bearing

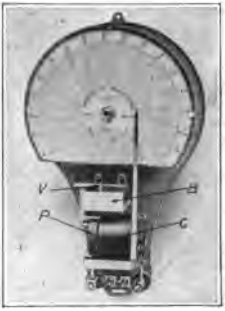


Figure 182.—Bristol Recording Wattmeter

knife-edge support at the right-hand end. With this arrangement, the two coils may be placed very close to each other and the desired movement of the upper end of the pen arm will be produced by a very small movement of the movable coil, making it possible with alternating-current instruments to obtain an evenly divided scale, as the magnetic field is quite constant over the short distance that the coil moves.

When the quantity being recorded rapidly fluctuates in value, it is desirable to have some form of damping device. The damping device provided consists of a light vane *V* attached to the movable element and moving in a box *B* filled with oil.

*General Electric Recording Instruments.*—The recording voltmeters, wattmeters, and power-factor meters manufactured by the General Electric Company operate on the direct-reading dynamometer principle. The frequency meter is of the tuned circuit type, and the ammeters are constructed on the magnetic vane principle. The voltmeter and the single-phase wattmeters have one fixed and one movable set of measuring coils; the polyphase wattmeters have two sets of fixed coils and two sets of movable coils, the latter being rigidly attached to a common shaft. The power-factor instruments have two pressure coils and one current coil, so arranged as to operate the recording needle and give a continuous record of the true power factor of balanced two- or three-phase, three- or four-wire circuits.

The general appearance of all of these meters is practically the same and a front view of the polyphase wattmeter, with the cover removed, is shown in Figure 183. The moving element of these instruments is suspended entirely from the top by means of a steel piano wire. The lower end is accurately centered by a small steel pivot passing through a ringstone sapphire jewel. With this method of suspension, the friction of the moving system is small in comparison to the torque tending to produce a deflection. The moving system is held in its zero position by means of a spiral spring. It is connected to the pen by means of an arm attached to the upper end of the shaft of the moving system and extending toward the front of the instrument, the outer end of the arm being attached to the pen proper by an intermediate link which is hinged to the end of the arm in such a manner that it can move only in a horizontal plane. As a result of this arrangement, the pen moves in a straight line across the record paper instead of in a curve. The record paper is moved lengthwise under the pen by means of a clockwork, the standard rate being 3 inches per hour. The motion of the moving system is damped by means of metal disks rotating between the poles of permanent magnets.

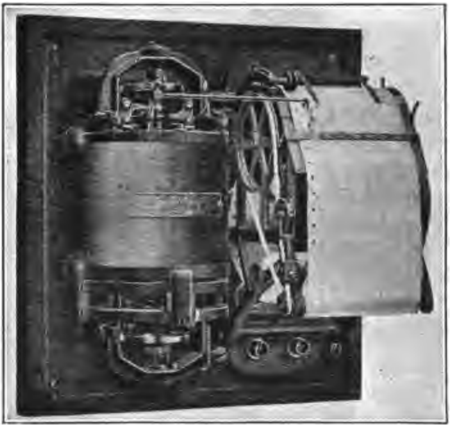


**Figure 183.—General Electric Polyphase Recording Wattmeter.**

*Westinghouse Recording Instruments.*—The recording voltmeters, alternating-current ammeters, wattmeters, and frequency meters, manufactured by the

Westinghouse Electric and Manufacturing Company, operate on the electrodynamometer or Kelvin-balance principle; the power-factor meter operates on the magnetic-vane principle; and the direct-current ammeters operate on the principle of D'Arsonval instruments. The ammeter is constructed with two coils, astatically arranged and pivoted within the magnetic fields of two permanent magnets, which reduces the effect of the earth's or any other external magnetic field.

*Westinghouse Recording Voltmeter.*—A front view of the recording voltmeter, with cover removed, is shown in Figure 184, and a schematic diagram of the electrical connections is shown in Figure 185. The four stationary coils *A, B, C, D,* and the two movable coils *E* and *F* are all connected in series and their terminals connected to the binding posts *3* and *4.* The

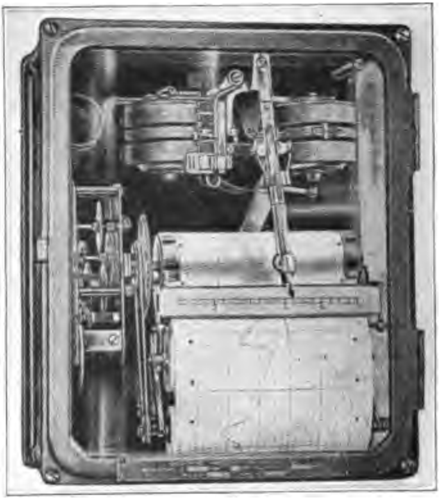


Figure 184.—Front View of Westinghouse Recording Voltmeter.

solenoid coils *K* and *L* are connected to the stationary relay contacts *H* and *I*, respectively, as shown, with their junction brought out to the binding post *2*. Movable coil *E* carries a contact *J*, which is electrically connected to binding posts *1*. Leads from the control circuit are connected to binding posts *1* and *2*, and leads from the circuit to be metered are connected to the binding posts *3* and *4*.

The recording element comprises the pen-actuating solenoids *K* and *L;* the iron cores *K′* and *L′*, which are supported by the T-shaped lever arm *M*, pivoted at *N;* the pen arm *O*, connected to *M* by pin bearing *P* and provided at the upper end with a pin *R* which moves in the stationary guide slot *V*; and the recording pen *S*, arranged to pass across a suitable record paper *T*, moved by clockwork not shown in the diagram.

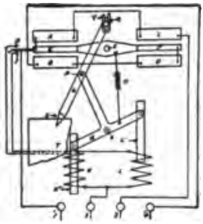


**Figure 185.—Diagram of Westinghouse Recording Voltmeter.**

The connections of the six coils *A, B, C, D, E,* and *F* are such that when there is a current through them, the coil *E* is repelled by *A* and attracted by *B*, while at the same time coil *F* is repelled by *D* and attracted by *C*. If the contact *J* is midway between the contacts *H* and *I*, and the recording pen *S* is at zero position on the record paper when connection is made to the solenoids and meter circuits, the contact *J* will be forced down against the contact *I*, thus completing the circuit through the solenoid *L*, energizing the latter and causing it to pull its core *L′* downward, thereby turning the arm *M* about the pivot *N*. This movement of *M* swings the pen arm *O* to the right, causing the latter to move across the record paper toward full-scale position. It will be observed that this movement of *M* also places the control spring *U* under a tension which continues to increase with the downward movement of the solenoid core *L′* until it balances the torque of the movable coils *E* and *F*, draws the contact *J* away from *I*, and breaks the circuit through the solenoid *L*. The dimensions and weights of the various moving parts of the meter and control springs are so proportioned that the entire moving system remains stationary or in the position occupied when the solenoid circuit was broken. The record paper is being drawn forward all the time, causing the pen to draw a line lengthwise, representing the voltage being maintained on the metered circuit.

If the pressure acting on the metered circuit should increase, the contact *J* is again forced down against the contact *I*, and the operation described above is repeated until the increased tension of the control spring *U* again balances the torque of the movable coils *E* and *F*, after which the moving system again remains stationary until there is another change in the pressure acting on the metered circuit.

When the pressure of the metered circuit decreases, the contact *J* is forced upward against the stationary contact *H*, thereby completing the circuit through the solenoid *K*. The resulting downward movement of core *K′* turns the supporting arm *M* to the left, causing the pen to be moved across the record paper toward the left or minimum-scale position. This movement continues until the tension in the spring *U* is sufficiently relieved to restore a balance between the actuating forces of the meter element and the spring. When this balance is obtained, the contact

*J* will leave the contact *H,* thus breaking the circuit through the solenoid *K.*

The distances *PN, PS,* and *PR* are made equal, which results in the lower end of the arm *O* moving across the record paper in a horizontal straight line. The pin *R,* which is rigidly connected to the upper end of the arm *O,* moves up and down in the guide slot *V.*

The moving parts of the meter and the recording elements are damped by means of suitably arranged pistons working in glycerine dashpots.

The character of the record made by the meter may be readily changed to suit various purposes by varying the distances between the stationary contacts *H* and *I.* When the contacts are placed close together, the pen will draw a curve showing the slightest variation in the metered quantity; and a more regular curve showing only the larger fluctuations in the metered quantity may be obtained by increasing the distance between the contact.

## MAXIMUM-DEMAND INSTRUMENTS

*Rates for Electrical Energy.*—Electrical energy can be supplied to the consumer most economically when generated at the time the consumer requires it, and the cost depends upon the amount supplied and also upon the time and the rate of supplying it. In order to be perfectly fair to both the consumer and the central station company, the various methods of charging should take the above facts into account. It is perfectly obvious, in general, that the cost of supplying a consumer with energy who uses power continuously for twelve hours per day will be less than the cost supplying a consumer with the same amount of energy per day who uses six times the power for only two hours per day. Less station capacity will be required in the first case and also smaller feeder circuits, both of which means a smaller investment for the central station company. Numerous systems of charging for electrical energy have been devised, and in the majority of cases they try to properly proportion the different rates in accordance with the cost of supplying electrical energy under different service conditions.

Two distinct types of instruments have been devised for the purpose of making the charge of electrical energy more equitable. These are the two-rate meter and the maximum-demand indicator.

*Two-Rate Watt-Hour Meters.*—An ordinary watt-hour meter may be equipped with two sets of dials and, by means of an electrical control operated by a time switch, one set of dials may be connected during certain hours of the day and the other set for the remaining hours, so that higher rates may be charged for energy used during certain hours of the day corresponding to maximum station load than is charged for energy used during the remainder of the day.

Two-rate meters are also made with a resistance in series with the armature which may be connected or disconnected by a time switch and the rate at which the meter element rotates changed. With this type of meter, only one set of dials is required, as the rate is automatically changed by changing the speed of the meter. When meters of this type are installed, the consumer does not, as a rule, use electrical energy as extravagantly during heavy load as at other times and the central station company can take care of the maximum load on the system more economically.

*Maximum-Demand Indicators.*—Maximum-demand indicators are instruments used in determining the maximum energy, power, or current used by a consumer in cases where the charge for energy is based upon the ratio of consumption to maximum demand. These instruments may be classified into three groups, depending upon the principle upon which they operate, viz., thermal, induction, and mechanical. A second classification into three groups may be made, based upon the nature of the indication or record, viz., instantaneous maximum-demand, time-lagged maximum-demand, and interval maximum-demand instruments.

In order to understand the purpose of these different types of maximum demand devices, it will be necessary that certain terms used in the measurement of demand be defined, and a few of the more common terms are usually defined as follows:

*Demand.*—The demand is a load that is specified, contracted for, or connected to the system for a specified time and is expressed in kilowatts, kilovolt-amperes, amperes, or other units. The load referred to is at the end of a line, feeder, etc., and the actual load on the station is greater than this by the losses in transmission.

*Maximum Demand.*—The maximum demand is the greatest demand used during a given time, such as a day, month, or year.

*Instantaneous Maximum Demand.*—The instantaneous maximum demand is the highest value of the demand, as determined by an instrument which indicates or records the true value of the demand at each instant.

*Time-Lagged Maximum Demand.*—The time-lagged maximum demand is the highest value of the demand, as determined by an instrument which requires a considerable interval of time to record the full value. This interval of time may be very small and in some cases as much as 30 minutes.

*Interval Maximum Demand.*—The interval maximum demand is the highest value of demand integrated over a given predetermined time.

*Simultaneous Maximum Demand.*—The simultaneous maximum demand is the greatest sum of the individual demands occurring at the same time.

*Demand Factor.*—The demand factor is the ratio of the maximum demand on any central station, system, or part of a system, to the total connected load of the central station, system, or part of a system under consideration.

*Load Factor.*—The load factor of a station, system, or part of a system is the ratio of the average load to the maximum load during any specified interval of time. The average load may be taken for a certain period, such as a month or a year, and the maximum load is taken within this same time.

*Instantaneous Demand Instruments.*—An instantaneous demand indicator is an instrument which will give an indication or record of the successive instantaneous demands. Thus, an indicating or recording wattmeter will give the values of the instantaneous demands. No record is made in the case of the indicating instruments and they are used only in special tests.

*Time-Lagged Demand Instruments.*—Time-lagged maximum-demand indicators do not give an indication or record of the instantaneous maximum demand but require a continuance of the load for a considerable time before the indicator registers the full value. Time-lagged maximum-demand indicators are made for indicating the maximum current or power demand.

*Wright Demand Indicator.*—The Wright demand indicator was one of the first instruments used in measuring the maximum load. Its operation depends upon the heating effect of the metered current, and its scale may be marked to read either the maximum current or the power, if the pressure of the circuit is constant. A front view of the instrument is shown in Figure 186, and a view of the registering element in Figure 187.

The instrument consists of a U-shaped glass tube *T* with a glass bulb at each end. A small overflow tube *O* branches from the right-hand side beneath the bulb and extends downward between the two limbs of the main tube. The U portion of the tube is filled with sulphuric acid which has been colored to make it more visible, and the bulbs are filled with air. A heating element is placed about or adjacent to the left-hand bulb, and terminals are provided so that this heating element may be connected in series with the circuit in which the current is to be measured. The heat generated by the current in the heating element causes the air in the left-hand bulb to expand,

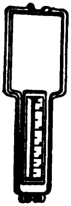


Figure 186.—Wright Demand Indicator.

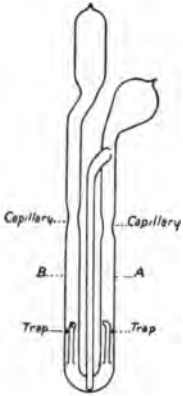


Figure 187.—Registering Element of Wright Demand Indicator.

and the liquid in that side of the tube is forced down and the liquid in the right-hand branch is forced up and passes into the over-flow tube. The scale of the instrument is placed behind this overflow tube and may be marked to read either current or power.

This instrument may be used on either direct- or alternating-current circuits, but it cannot be used in determining the demand in watts on the alternating-current circuit unless the power factor is unity or its value is known and is constant.

The instrument is slow acting and, on a steady load, will record about 90 per cent of it in 4 minutes and the full load in from 30 to 60 minutes. This instrument does not give the time of the maximum demand, and its record is destroyed after each reading.

*Westinghouse Watt-Hour Demand Meter.*—The Westinghouse watt-hour demand meter is a combination of induction watt-hour meter, an indicating induction wattmeter, and an escapement form of time element. The maximum demand is indicated directly in kilowatts by a pointer sweeping over a four-inch dial, and the total watt-hour load on a four-dial counter. The appearance of the glass-covered meter is shown in Figure 188.

The instrument has a standard watt-hour meter movement, including electromagnet, permanent magnets, and aluminum disk, and, in addition, an auxiliary disk sector supported so that it can move in the air gap of the electromagnet in such a way that it does not interfere with the accuracy of the main disk, which always rotates at a speed proportional to the load. The rotation of the auxiliary disk is restrained by a spiral spring and its final deflection is proportional to the watts load. The auxiliary disk with its spring and pointer constitutes an induction indicating wattmeter.

The shaft of the auxiliary disk is geared to an escapement wheel, and the claw restricting this wheel is oscillated by an eccentric on the main shaft. The auxiliary disk, therefore, advances step by step at a speed determined by the speed of the main watt-hour meter disk, that is, at a speed proportional to the load. It continues to advance until the torque in the spring is sufficient to balance the torque produced in the auxiliary disk. The auxiliary disk drives the maximum demand pointer through a dog, and a fine-toothed ratchet and pawl hold the demand pointer in the position of maximum deflection until released by hand. A second ratchet and pawl allow the auxiliary disk to fall back to equilibrium under its spring tension if the load falls below that corresponding to its position at any moment, but prevent it from advancing except as controlled by the escapement as before. The maximum demand pointer can be reset instantly by pressing a sealed button on top of the cover.

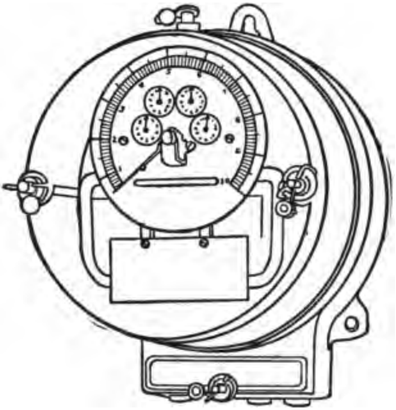


**Figure 188.—Westinghouse Watt-Hour Demand Meter.**

The meters are constructed to indicate the average maximum demand over 1-, 2-, 5-, 15-, and 30-minute time intervals. The meter is direct reading but, as in the case of the Wright demand indicator, the time of maximum demand is not given. It has the advantage of combining the maximum demand indicator and watt-hour meter in one, thus reducing the initial cost and also the cost of installation.

*Interval Maximum-Demand Instruments.*—Interval maximum-demand instruments are so constructed that they show the integrated value of the maximum load over a definite interval of time. They may be grouped into two general classes: those indicating the maximum load only, and those indicating the values of the demand during successive intervals of time and also the time at which they occur.

*Maxicator.*—The maxicator is a form of interval maximum-demand instrument, giving an indication of the interval maximum demand during any period. A front view of the maxicator is shown in Figure 189. It is exactly similar to the ordinary watt-hour meter register except it has added thereto a separate train of gears carrying a maximum-demand pointer. Maxicators are designed to be mounted in place of the standard meter registers, since they give the total consumption in watt-hours and the maximum demand reading.

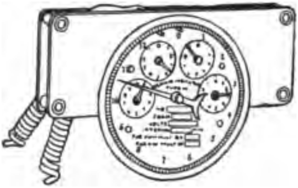


Figure 189.—Front View of Maxicator.

The four small dial hands correspond to the dial hands of the ordinary watt-hour meter register, which give a reading of the total consumption. The large center dial hand is known as the maximum-demand pointer and indicates on the large external scale the measured maximum demand during 30- or 15-minute intervals over any period, such, for instance, as a week or a month. This maximum-demand pointer is merely a friction pointer and is not connected directly in the train of gears.

The register of the maxicator operates as an ordinary watt-hour meter and has connected with it through a train of gears a driving element that engages the maximum-demand pointer. Therefore, as the meter progresses, this driving element will be carried forward, and, in turn, will carry forward the maximum-demand pointer at the same rate of speed and, of course, at a speed directly proportional to the consumption of energy passing through the watt-hour meter. This driving will be continued until the end of the time interval, when a solenoid connected with the driving element will be energized by means of a contact-making motor or contact-making clock, and will reset the driving element to zero position but leave the maximum-demand pointer in its position on the large scale, it being held in position by a light spring bearing against its shaft.

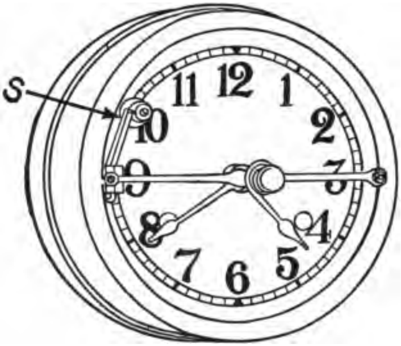


Figure 190.—Contact-Making Clock.

Immediately after the driving element has been reset to zero position, it will begin to revolve as before, but, of course, will not re-engage and drive the maximum-demand pointer to a higher point unless the load on the meter increases and the demand for the next interval is greater than the preceding. The position to which the maximum-demand pointer is finally carried and left will be an indication of the greatest number of revolutions of the meter disk during the time interval and, therefore, an indication of the largest consumption of energy that has passed through the meter during the interval of time for which the maxicator is arranged. The time at which this maximum demand occurred is not shown by the maxicator.

A front view of the contact-making clock used with the maxicator is shown in Figure 190. A cam short-circuits two contact springs at *S*, thus closing the circuit of the maxicator set-back solenoid. The cam is not operated by the spring of the clock proper, but by a second independent mainspring, which prevents any disturbance of the timing movement.

*Printometer.*—The printometer is a form of interval maximum-demand instrument which prints in plain figures on a piece of paper tape the maximum demand during successive intervals of time and also the time during which the consumption occurred. The front of the instrument is shown in Figure 191. This instrument contains a set of cyclometer-type wheels which are electrically interlocked with the register of the watt-hour meter. These wheels are moved forward by the action of electromagnets which become energized when a circuit is closed by a contact commutator placed on the spindle of the watt-hour meter. They are moved forward at a rate which is exactly equal to the rate of flow of power through the watt-hour meter and will, therefore, at any instant give an indication which is equivalent to the reading of the meter dial. Through the agency of a rubber plate and a copying ribbon, this reading is printed on a paper tape. When the printing solenoid is energized at recurring intervals by means of a contact-making device, there is obtained a paper tape which has printed upon it a reading equivalent to the reading of the watt-hour meter at the end of each interval of time.

Printometers are quite expensive to install and maintain, but they are quite extensively used in the sale of energy to large consumers.

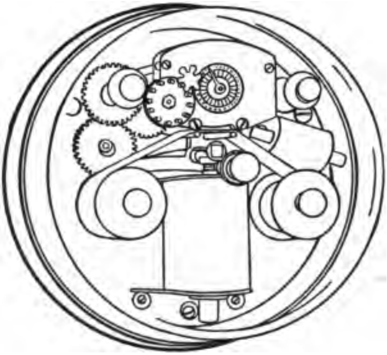


Figure 191.—Front View of Printometer.

*Graphometers.*—The graphometer is an instrument to be used in connection with watt-hour meters. It gives a graphic record which shows the average demand during each of the time intervals, and also the time of day and day of the week, when such demand occurred.

The record of the graphometer differs from that of the other types of graphic recording instruments in that it gives the kilowatt hours registered by the watt-hour meter during each interval of time, while the other types give the instantaneous values of the power.

# APPENDIX

## TABLE III

## COPPER WIRE TABLE

### Working Table, International Standard Annealed Copper

### American Wire Gage (B. & S.)

| Gage No. | Diameter in Mils | Cross Section |  | Ohms per 1000 Feet |  | Pounds per 1000 Feet |
|---|---|---|---|---|---|---|
|  |  | Circular Mils | Square Inches | 25° C (=77° F) | 65° C (=149° F) |  |
| 0000 | 460. | 212 000. | 0.166 | 0.0500 | 0.0577 | 641. |
| 000 | 410. | 168 000. | .132 | .0630 | .0727 | 508. |
| 00 | 365. | 133 000. | .105 | .0795 | .0917 | 403. |
| 0 | 325. | 106 000. | .0829 | .100 | .116 | 319. |
| 1 | 289. | 83 700. | .0657 | .126 | .146 | 253. |
| 2 | 258. | 66 400. | .0521 | .159 | .184 | 201. |
| 3 | 229. | 52 600. | .0413 | .201 | .232 | 159. |
| 4 | 204. | 41 700. | .0328 | .253 | .292 | 126. |
| 5 | 182. | 33 100. | .0260 | .319 | .369 | 100. |
| 6 | 162. | 26 300. | .0206 | .403 | .465 | 79.5 |
| 7 | 144. | 20 800. | .0164 | .508 | .586 | 63.0 |
| 8 | 128. | 16 500. | .0130 | .641 | .739 | 50.0 |
| 9 | 114. | 13 100. | .0103 | .808 | .932 | 39.6 |
| 10 | 102. | 10 400. | .008 15 | 1.02 | 1.18 | 31.4 |
| 11 | 91. | 8230. | .006 47 | 1.28 | 1.48 | 24.9 |
| 12 | 81. | 6530. | .005 13 | 1.62 | 1.87 | 19.8 |
| 13 | 72. | 5180. | .004 07 | 2.04 | 2.36 | 15.7 |
| 14 | 64. | 4110. | .003 23 | 2.58 | 2.97 | 12.4 |
| 15 | 57. | 3260. | .002 56 | 3.25 | 3.75 | 9.86 |
| 16 | 51. | 2580. | .002 03 | 4.09 | 4.73 | 7.82 |
| 17 | 45. | 2050. | .001 61 | 5.16 | 5.96 | 6.20 |
| 18 | 40. | 1620. | .001 28 | 6.51 | 7.51 | 4.92 |
| 19 | 36. | 1290. | .001 01 | 8.21 | 9.48 | 3.90 |
| 20 | 32. | 1020. | .000 802 | 10.4 | 11.9 | 3.09 |

| Gage No. | Diameter in Mils | Cross Section | | Ohms per 1000 Feet | | Pounds per 1000 Feet |
|---|---|---|---|---|---|---|
| | | Circular Mils | Square Inches | 25° C (=77° F) | 65° C (=149° F) | |
| 21 | 28.5 | 810. | .000 636 | 13.1 | 15.1 | 2.45 |
| 22 | 25.3 | 642. | .000 505 | 16.5 | 19.0 | 1.94 |
| 23 | 22.6 | 509. | .000 400 | 20.8 | 24.0 | 1.54 |
| 24 | 20.1 | 404. | .000 317 | 26.2 | 30.2 | 1.22 |
| 25 | 17.9 | 320. | .000 252 | 33.0 | 38.1 | 0.970 |
| 26 | 15.9 | 254. | .000 200 | 41.6 | 48.0 | .769 |
| 27 | 14.2 | 202. | .000 158 | 52.5 | 60.6 | .610 |
| 28 | 12.6 | 160. | .000 126 | 66.2 | 76.4 | .484 |
| 29 | 11.3 | 127. | .000 099 5 | 83.4 | 96.3 | .384 |
| 30 | 10.0 | 101. | .000 078 9 | 105. | 121. | .304 |
| 31 | 8.9 | 79.7 | .000 062 6 | 133. | 153. | .241 |
| 32 | 8.0 | 63.2 | .000 049 6 | 167. | 193. | .191 |
| 33 | 7.1 | 50.1 | .000 039 4 | 211. | 243. | .152 |
| 34 | 6.3 | 39.8 | .000 031 2 | 266. | 307. | .120 |
| 35 | 5.6 | 31.5 | .000 024 8 | 335. | 387. | .0954 |
| 36 | 5.0 | 25.0 | .000 019 6 | 423. | 488. | .0757 |
| 37 | 4.5 | 19.8 | .000 015 6 | 533. | 616. | .0600 |
| 38 | 4.0 | 15.7 | .000 012 3 | 673. | 776. | .0476 |
| 39 | 3.5 | 12.5 | .000 009 8 | 848. | 979. | .0377 |
| 40 | 3.1 | 9.9 | .000 007 8 | 1070. | 1230. | .0299 |

NOTE—The table is based on the international standard of resistance for copper, which takes the fundamental mass resistivity = 0.15328 ohm (meter, gram) at 20° C, the corresponding temperature coefficient = 0.00393 at 20° C, and the density = 8.89 grams per cc at 20° C. The temperature coefficient is proportional to the conductivity, whence the change of mass resistivity per degree C is a constant, 0.000597 ohm (meter, gram).

NOTE 2—The values given in the table are only for annealed copper of the standard resistivity. The user of the table must apply the proper correction for copper of any other resistivity. Hard-drawn copper may be taken as about 2.7 per cent higher resistivity than annealed copper.

NOTE 3—Ohms per mile, or pounds per mile, may be obtained by multiplying the respective values above by 5.28.

NOTE 4—For complete tables and other data see Circular No. 31 of the Bureau of Standards.

## TABLE IV

### SPECIFIC RESISTANCE, TEMPERATURE COEFFICIENT, PERCENT RELATIVE RESISTANCE AND CONDUCTANCE OF DIFFERENT MATERIALS

| Material | Measurements at 0° Centigrade | | | Relative Resistance % | Relative Cond % | Temperature Coefficient | |
|---|---|---|---|---|---|---|---|
| | Michroms per cub. cm | Michroms per cub. inch | Mil — foot resistance | | | Centigrade | Fahrenheit |
| Copper (Matthiessens Standard) | 1.594 | .6272 | 9.54 | 100.0 | 100.0 | .00420 | .00233 |
| Copper Annealed | 1.56 | .614 | 9.35 | 97.5 | 102.6 | .00428 | .00242 |
| Silver | 1.47 | .579 | 8.82 | 92.5 | 108.2 | .00400 | .00222 |
| Zinc (pure) | 5.75 | 2.26 | 34.5 | 362. | 27.6 | .00406 | .00226 |
| Iron (very pure) | 9.07 | 3.57 | 54.5 | 570. | 17.6 | .00625 | .00347 |
| Lead (pure) | 20.4 | 8.04 | 123. | 1280. | 7.82 | .00411 | .00228 |
| Platinum (annealed) | 8.98 | 3.53 | 53.9 | 565. | 17.17 | .00247 | .00137 |
| Mercury | 94.3 | 37.1 | 566. | 5930. | 1.69 | .00072 | .00044 |
| Gold (practically pure) | 2.2 | .865 | 13.2 | 138. | 72.5 | .00377 | .00210 |
| Aluminum (99% pure) | 2.6 | 1.01 | 15.4 | 161. | 62.1 | .00423 | .00235 |

## TABLE V

### ALLOWABLE CURRENT CARRYING CAPACITIES OF WIRES

| B. & S. G. | Rubber Insulation Amperes | Other Insulation Amperes | Circular Mils |
|---|---|---|---|
| 18 | 3 | 5 | 1,624 |
| 16 | 6 | 10 | 2,583 |
| 14 | 15 | 20 | 4,107 |
| 12 | 20 | 25 | 6,530 |
| 10 | 25 | 30 | 10,380 |
| 8 | 35 | 50 | 16,510 |
| 6 | 50 | 70 | 26,250 |
| 5 | 55 | 80 | 33,100 |
| 4 | 70 | 90 | 41,740 |
| 3 | 80 | 100 | 52,630 |
| 2 | 90 | 125 | 66,370 |
| 1 | 100 | 150 | 83,390 |
| 0 | 125 | 200 | 105,500 |
| 00 | 150 | 225 | 133,100 |
| 000 | 175 | 275 | 167,800 |
| 0000 | 225 | 325 | 211,600 |
| Circular Mils | | | |
| 200,000 | 200 | 300 | |
| 300,000 | 275 | 400 | |
| 400,000 | 325 | 500 | |
| 500,000 | 400 | 600 | |
| 600,000 | 450 | 680 | |
| 700,000 | 500 | 760 | |
| 800,000 | 550 | 840 | |
| 900,000 | 600 | 920 | |
| 1,000,000 | 650 | 1,000 | |
| 1,100,000 | 690 | 1,080 | |
| 1,200,000 | 730 | 1,150 | |
| 1,300,000 | 770 | 1,220 | |
| 1,400,000 | 810 | 1,290 | |
| 1,500,000 | 850 | 1,360 | |
| 1,600,000 | 890 | 1,430 | |
| 1,700,000 | 930 | 1,490 | |
| 1,800,000 | 970 | 1,550 | |
| 1,900,000 | 1,010 | 1,610 | |
| 2,000,000 | 1,050 | 1,670 | |

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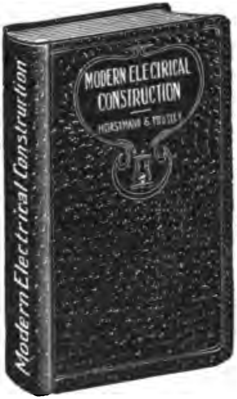
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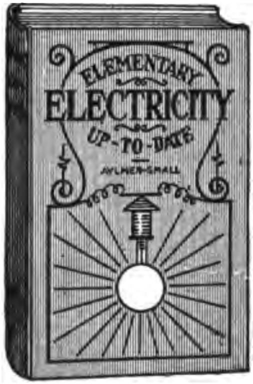

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